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Powder Technology 182 (2008) 313–322
POWDER TECHNOLOGY
www.elsevier.com/locate/powtec

A model of non-Newtonian slurry flow in a fracture

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Received 17 February 2007; received in revised form 19 June 2007; accepted 27 June 2007
Available online 3 July 2007

Abstract

An accurate calculation of a non-Newtonian slurry flow in a fracture is an important issue for fracture design (see for example, the book edited by Economides and Nolte [M.J. Economides, K.J. Nolte, Reservoir Stimulation, Third edition, Schlumberger, 2000.]). A model taking into account micro-level particle dynamics is developed here. The model shows that the slurry dynamics is governed to a significant extent by particle fluctuations about mean streamlines in a high-shear-rate flow. Particles migrate from zones of high shear rate at the fracture walls towards the center of the fracture where shear rates are lower. Thus, slurry flow in a fracture is characterized by non-uniform solids concentration across the fracture width. Low solids concentration near the walls leads to a reduction of slurry-wall friction as compared with that predicted by a model that does not take particle migration into account. Reduction in the friction at the wall leads to a reduction in the streamwise pressure gradient and hence in the net pressure.

Keywords: Granular temperature; Hydraulic fracturing; Net pressure; Non-Newtonian slurry; Particle migration; Proppant

1. Introduction

Hydraulic fracturing is an important technology for stimulating wells to increase the rate of production of oil or gas. The principle is simple. A slurry consisting of a viscous fluid (usually an aqueous solution of a natural polymer such as guar) and near-spherical particles (proppant or sand) is pumped into a cased wellbore at high pressure. It flows into the reservoir through perforations in the casing creating a large planar fracture, provided that the fluid pressure at the perforations is sufficiently high to overcome the least principal earth stress (the closure pressure) in the reservoir. As slurry flows along the fracture the fracture widens and propagates; frictional (net) pressure drop in the flowing slurry means that the fluid pressure decreases from perforations to the fracture tip. The geometry of a growing fracture is dependent on this pressure distribution. When pumping ceases, most of the slurry fluid left in the fracture leaks off into the reservoir rock bordering the fracture walls, and the fracture closes onto the proppant left in the fracture. This leaves a narrow permeable channel (typically about 10 mm wide) between the walls of the fracture (which can be several hundred meters long and is typically 50 to 100 m high). Fig. 1 shows a horizontal section of a typical fracture. The propped fracture is much more permeable than the reservoir rock around it because the proppant particles are much larger than the grains of the rock. This large increase in effective reservoir conductivity near the wellbore leads to a large increase in well productivity.

Details about the modeling and design of hydraulic fractures can be found in [1]. Various computational codes have been developed for the necessary calculations. The predictions made are only as good as the models they use. A disappointingly high proportion (30%) of fracturing jobs fail in one sense or another. This paper provides an improved model for calculating the transport of proppant and associated pressure gradients within the fracture, consistent with the framework presented in [2].

2. Model

Slurry flows moving in fractures during fracturing procedures are characterized by high mean shear rates (up to $\dot{\gamma}_m = u_m / w = 200$ 1/s), where u_m is the mean (superficial) slurry velocity and w is the fracture width (see Fig. 2). In our analysis we will

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doi:10.1016/j.powtec.2007.06.027

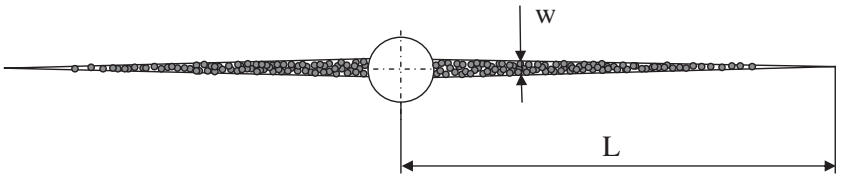

Fig. 1. Schematic of a hydraulic fracture in a plane perpendicular to the wellbore.

consider a steady-state slurry flow in a channel of a constant width. We do not consider the leak-off impact on slurry dynamics. This assumption corresponds to the real situation when the formation permeability is relatively low. It is also important to note that the bigger the proppant the higher the fracture conductivity; therefore, in industry there is a tendency to increase particle size (up to about 2000 μm in practical applications).

Many fracturing fluids used in practice are characterized by power-law rheology. Engineers usually employ a known solution of the Navier–Stokes equation for power-law slurry [1] obtained at assuming that particles have the same velocity as a fluid and that the concentration of solids is uniformly distributed across a fracture. Lattice–Bolzmann computations have shown that in a high-shear-rate slurry flow particles fluctuate [3].

For the present study, the kinetic theory of granular flows [4] was employed for flow modeling. The kinetic theory assumes that particles collide with each other moving like ideal gas molecules, and the velocity distribution of particle fluctuations is Maxwellian. The key parameter of this approach is the granular temperature, which determines the intensity of particle fluctuations:

$$\Theta = \frac{\langle v_s^2 \rangle}{3}, \tag{1}$$

where $\langle v_s^2 \rangle$ is the mean-square fluctuation velocity of a particle.

In contrast to the successful modeling of gas-particles flows using the kinetic theory, the modeling of liquid–solid flows using the kinetic theory is still a challenge.

Nott and Brady [5] used the kinetic approach for modeling particle migration across a flat channel in a Newtonian slurry flow. They considered the Stokesian regime of particle–fluid interactions characterized by zero particle Reynolds numbers in particle–fluid relative motion. Effects caused by the influence of neighboring particles on pair particle–particle interactions at high solids concentrations were not taken into account. Nevertheless, the results obtained by that model were in a good agreement with the data obtained by the numerical calculations based on the Stokesian dynamics. Note that Nott and Brady showed also that the earlier model developed by Leighton and Acrivos [6], which did not involve particle fluctuations in the analysis, demonstrated poorer performance than the model based on the kinetic approach.

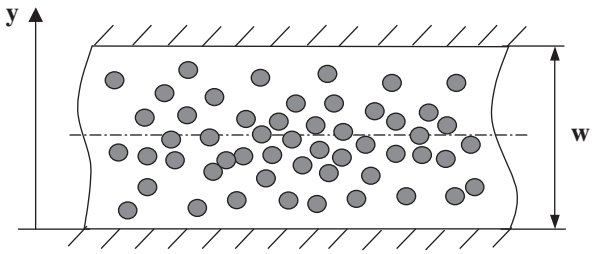

Fig. 2. Schematic of a channel representing a fracture in a model.

According to Wylie et al. [3], the kinetic theory can be reliably applied to modeling of fluid–solids flows if the ratio of the velocity relaxation time for a particle fluctuating in a liquid to the time of a particle free path between particle–particle collisions is much higher than unity. Based on the lattice-Bolzmann computations Wylie et al. [3] showed that the kinetic theory is accurate if the aforementioned ratio is larger than 2. Validity of the kinetic theory application to the slurry flow in a fracture is discussed below (see "Numerical examples and discussion" and "Model validity and practical applications").

Viscosity of a carrying fluid is usually very high and has shear-thinning rheological properties. For example, depending on the shear rate, the viscosity of a fracturing fluid can be two to four orders of magnitude higher then the water viscosity. Additionally, the slurry viscosity is higher than that of a pure liquid because of particle–fluid viscous interaction. There are a number of experimental and theoretical equations predicting slurry viscosity increase with increasing the solids concentration [7]. Note that within the original kinetic theory developed mainly for gas–particle flows, the transport equations for gas and solids phases are formulated separately. The separate viscosity of a solids phase is introduced in that case and modeled as caused by spatial particle momentum transfer in a particle-fluctuation motion like in a molecular gas [4]. Because in our research we model dense solids–liquid systems, in which particle–fluid interactions on micro-level are complicated, we formulate the transport equations using the fluid–solids mixture viscosity. There is, therefore, no need to introduce the separate viscosity of a solids phase. For calculation of the slurry viscosity increase with increase in the solids concentration we employed one of the known empirical correlations [7] determining the ratio of the slurry viscosity to the viscosity of carrying fluid:

$$f(c) = \frac{\mu_{sl}}{\mu_L} = 1 + 2.5c + 10c^2 + 0.0019\exp(20c), \tag{2}$$

where c is the volume solids concentration and μ_L and μ_{sl} are the dynamic fluid and slurry viscosities, respectively.