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Analytic solutions for filtration of polydisperse aerosols in fibrous filter

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Abstract

In this study, analytical solutions for penetration efficiency of a polydisperse aerosol in fibrous filter were derived employing Brownian diffusion and inertial impaction as removal mechanisms. Size distribution of aerosol particles was assumed to be represented by a log-normal function during the filtration. Derived solutions were compared with the exact solution, which is not based on the log-normal assumption, showing good agreement. Error resulting from the log-normal assumption was shown to be greater in the impaction-dominant regime than in the diffusion-dominant regime due to higher size dependency of collision kernel which destructed log-normal shape of size distribution. The penetration efficiency of the analytic solution initially decreases faster and then decreases slower than that of the exact solution in the diffusion-and intermediate dominant size regimes due to its polydispersity of particle distribution, while it overpredicted the particle removal in the impaction size range because of neglect of polidispersity effect. A new solution for the most penetrating particle diameter was also provided showing the dependence on filtration velocity, fiber volume fraction, and fiber size.

Keywords: Filtration; Moment method; Brownian diffusion; Inertial impaction

1. Introduction

Filtration by fibrous filters is one of the principal methods used for removing particles in the all size range. Fibrous filters are relatively inexpensive and are simple to operate, yet they provide the most efficient means of collecting particles. Because of the increasing need to protect both human health and valuable devices from exposure to fine particles, filtration has become more important. In the case of the filtration of very small particles, the effects of inertia, gravitational settling, and interception can be neglected and Brownian diffusion becomes the dominant mechanism for particle collection, whereas, in the case of the filtration of very large particles, the effects of inertia are more dominant mechanisms for particle collection. The single fiber efficiency due to diffusion and impaction is a function of flow velocity, the particle collision kernel, and the diameter of a fiber. If the flow velocity and the diameter of a fiber are fixed, the only variable is the collision kernel of the particle. Then the particle size of the size distribution of an aerosol becomes the dominant determining factor of single fiber efficiency since the collision kernel is a function of particle size. The effect of particle size distribution was considered first by Soderholm [1] and Lee et al. [2], who extended the screen-type diffusion battery theory of Cheng and Yeh [3] and Cheng et al. [4] to provide a more direct method for determining the mean particle size and deviation of log normal aerosol size distributions. Subsequently, the penetration of polydisperse aerosols in a screen-type diffusion battery was calculated numerically by Lee [5], who employed Brownian diffusion and interception as the applicable deposition mechanisms. In the case of fibrous filter filtration, Kim et al. [6] studied the diffusional filtration of polydisperse aerosol particles using a numerical method. Recently, an analytical solution was derived for the change in particle size distribution [7]. This solution, however, was valid only for small particles because inertial impaction was not accounted for. For the case where inertial impaction is the predominant removal mechanism, Friedlander and Pasceri [8] suggested another alternative formula for collection efficiency. Rosner et al. [9] verified the local size distributions of particles deposited by inertial impaction on a cylindrical target has retained log-normal shape when the mainstream particle suspension is log-normal.

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In this study, analytical solutions for filtration of a polydisperse aerosol in impaction dominant size regime as well as in diffusion dominant size regime are derived employing Brownian diffusion, interception and inertial impaction as removal mechanisms. Furthermore, a new solution for the most penetrating particle diameter is provided.

2. Governing equation

The removal of a polydisperse aerosol by filtration is represented by

$$\frac{\partial n(d_p,z)}{\partial z} = -n(d_p,z)\int_0^\infty \chi(d_p,D_f)n_f(D_f)\mathrm{d}D_f, \tag{1}$$

where $n(d_p,z)$ is the particle size distribution function in position in the filter z, d_p is the particle diameter, $n_f(D_f)$ is the fiber size distribution function, and D_f is the fiber diameter. The collision kernel χ is given as

$$\chi(d_p,D_f) = \frac{4\alpha}{\pi(1-\alpha)D_f}E(d_p,D_f), \tag{2}$$

where α is the packing density in the filter and $E(d_p,D_f)$ is the efficiency of collection between an aerosol particle d_p and a fiber diameter D_f.

According to Lee and Liu [10], neglecting minor terms and simplifying the equation, the collection efficiency due to Brownian diffusion is

$$E_{\mathrm{diff}}(d_p,D_f) = 2.6\left(\frac{1-\alpha}{K}\right)^{1/3}Pe^{-2/3}, \tag{3}$$

where $K = -\frac{1}{2}\ln\alpha - \frac{3}{4} + \alpha - \frac{1}{4}\alpha^2$ and Pe is the Peclet number defined as

$$Pe = \frac{D_f U}{D_{\mathrm{diff}}}, \tag{4}$$

D_{diff} is the diffusion coefficient of aerosol particles given by

$$D_{\mathrm{diff}} = \frac{k_B T C_c(d_p)}{3\pi\mu d_p}, \tag{5}$$

where k_B is the Boltzmann constant, T is the absolute temperature, μ is the air viscosity, and C_c is the Cunningham slip correction factor:

$$C_c = 1 + 2.493\frac{\lambda}{d_p} + 0.84\frac{\lambda}{d_p}\exp\left(-0.435\frac{d_p}{\lambda}\right), \tag{6}$$

where λ is the mean free path length of air molecules.

If a particle follows a gas streamline coming within one particle radius of a filter fiber, the particle is captured by the fiber, which is called interception. According to Lee and Liu [10], the collection efficiency due to interception is given by

$$E_{\mathrm{int}}(d_p,D_f) = 0.6\left(\frac{1-\alpha}{K}\right)\frac{R^2}{(1+R)}, \tag{7}$$

where $R = d_p/D_f$.

For large particles, inertial impaction is the dominant removal mechanism of filtration in fibrous filter. The dimensionless number that describes the inertial impaction property of a particle is the Stokes number defined as

$$\mathrm{Stk} = \frac{\rho_p d_p^2 U}{18\mu D_f}, \tag{8}$$

where ρ_p is the particle density. The collection efficiency due to inertial impaction is given by [11]

$$E_{\mathrm{imp}}(d_p,D_f) = \frac{\mathrm{Stk}^3}{(\mathrm{Stk}^3 + 0.77\mathrm{Stk}^2 + 0.22)}, \tag{9}$$

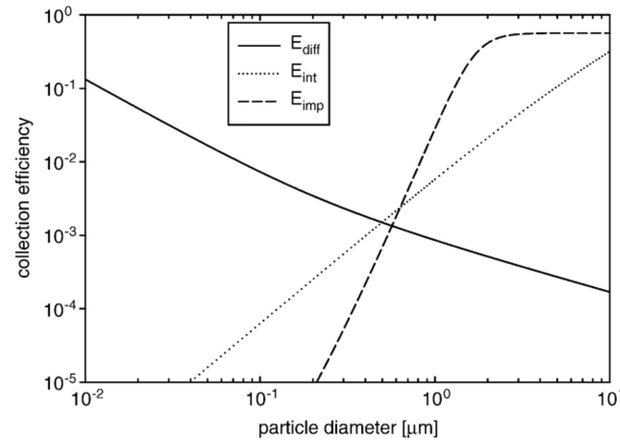

Fig. 1. Comparison of collection efficiency to Brownian diffusion, interception and inertial impaction (U=0.5 m/s, D_f=10 μm).

3. Approximate collection efficiency

The overall collection efficiency accounting for Brownian diffusion, interception and inertial impaction can be represented by the sum of Eqs. (3), (7), and (9). The resulting equation, however, is very complicated making it difficult to derive analytical solutions for particle size distribution change. Thus, we first introduce several approximations for the collection efficiency.

Fig. 1 compares the collection efficiencies due to the three mechanisms as a function of particle diameter. For this figure, the fiber size was set at 10 μm and the packing density is $5*10^{-2}$. It is shown that inertial impaction is dominant for large particles while Brownian diffusion dominants for small particles. When the filtration velocity, U, was 0.5 m/s, we examined this for different values of the fiber size (2 μm$<D_f<$100 μm) and the packing density ($10^{-4}<\alpha<0.1$), and confirmed that interception may always be neglected. From Fig. 1, we can notice there is a particle size for which neither Brownian diffusion nor inertial impaction actively removes particles.

Lee and Liu [12] used the following forms of the Cunningham slip correction factor based on the Knudsen–Weber equation:

$$C_c = 1.664\frac{2\lambda}{d_p} \quad \text{for Kn>2.6 or } d_p\text{<0.05 } \mu\text{m}, \tag{10}$$

$$C_c = 2.609\sqrt{\frac{2\lambda}{d_p}} \quad \text{for 0.15<Kn<2.6 or 0.05 } \mu\text{m} < d_p < 1.0\,\mu\text{m}, \tag{11}$$