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Elastohydrodynamic theory for wet oblique collisions

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Abstract

This paper presents a model for oblique collisions of spherical particles with a plane surface covered with a thin liquid layer. Elastohydrodynamic theory developed previously for fully immersed collisions [Davis, Serayssol and Hinch 1986 *JFM* 63 479–497] is modified for the normal component of motion to account for the finite thickness of the liquid layer. The resulting time evolution of the film thickness profile is then used along with sliding lubrication to determine the tangential component of motion. The critical Stokes number (dimensionless ratio of particle inertia and viscous forces), below which no rebound is seen, is predicted in terms of the physical properties of the materials involved in the collision, as described by a compliance parameter representing a dimensionless measure of elastic deformation due to viscous forces. Beyond the critical Stokes number, the normal restitution coefficient is found to increase with the Stokes number and the compliance parameter, asymptoting to the dry restitution coefficient at high Stokes numbers. The lubrication suction resistance during rebound is limited by cavitation. The tangential restitution is independent of the impact angle and is linearly dependent on the ratio of the fluid layer thickness to the sphere radius, in addition to depending on the Stokes number and compliance parameter. The tangential restitution is found to be close to unity and is generally higher for a larger value of the compliance parameter. Moreover, the tangential restitution is seen to increase with the Stokes number at small compliance and decrease with the Stokes number at large compliance. The change in rotational velocity exhibits trends that are the reverse of the tangential restitution. Finally, closed-form expressions have been developed for describing the restitution coefficients and dimensionless change in rotational velocity.

Keywords: Modeling; Restitution; Particle collisions; Elasticity; Hydrodynamics

1. Introduction

The problem of two spheres, or of one sphere and a flat surface, undergoing a head-on elastic collision under dry conditions was first studied by Hertz [1]. Thereafter, several studies have been published, including the *JKR* theory provided by Johnson, Kendall and Roberts [2], which considers adhesive forces in inelastic collisions, and the work of Maw, Barber and Fawcett [3], which is an extension of the Hertzian theory to oblique impacts. A review on the forces involved in contact modeling can be found in [4]. There have also been several experimental investigations of dry sphere-plane collisions (e.g., [5–9]) and sphere-sphere collisions (e.g., [10,11]).

Collisions of particles in the presence of liquids occur in several applications, including filtration, coagulation, agglomeration, inertial capture, etc. Wet collisions are much more complex than dry collisions, as they involve the coupling of fluid dynamics with solid mechanics. Davis, Serayssol and Hinch [12] developed elastohydrodynamic theory for studying head-on collisions between two spheres in close contact and immersed in a liquid. Their work provides the first rational criteria for predicting whether the spheres will stick or rebound after colliding in a liquid. This work also determines the dynamic force, relative velocity, separation and surface deformation profile of the spheres, based on two non-dimensional parameters. The first parameter is the Stokes number, which represents the ratio of inertia of the spheres and the viscous forces in the separating fluid layer:

$$St = \frac{mV_o}{6\pi\mu a^2}, \tag{1}$$

where m is the reduced mass of the two spheres, V_o is the initial relative normal velocity, μ is the viscosity of the fluid and a is

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the reduced radius of the two spheres. The second non-dimensional parameter is the elasticity parameter, which represents the ratio of the viscous forces tending to deform the solids and the stiffness of the solids resisting deformation:

$$\varepsilon = \frac{4\theta\mu V_o a^{3/2}}{x_o^{5/2}}, \tag{2}$$

where x_o is the initial separation between the undeformed spheres and

$$\theta = \frac{1-v_1^2}{\pi\ E_1} + \frac{1-v_2^2}{\pi\ E_2} \tag{3}$$

is a function of material properties, with v_i and E_i being Poisson's ratio and Young's modulus for sphere i, respectively. The special case of a sphere impacting a plane results when $a = a_1$ and $a_2 \to \infty$. The restitution coefficient, e, is defined as the ratio of the magnitude of the relative rebound velocity to the relative approach velocity. Davis *et al*. [12] considered the ratio of the maximum rebound velocity to the initial approach velocity as the restitution coefficient. They presented a plot of the restitution coefficient versus Stokes number for various elasticity parameters.

Davis [13] extended the theoretical work of Davis *et al*. [12] to account for surface roughness, adhesive forces between particles, and the discrete molecular nature of the fluid. Barnocky and Davis [14] dropped spheres onto both smooth and rough surfaces covered with thin liquid layers from varying heights and observed the critical drop height at which the spheres stopped sticking and above which rebound from the surface occurred. Their results are in agreement with the elastohydrodynamic theory, with approximate corrections to account for cavitation and the finite thickness of the liquid layer. Lundberg and Shen [15] studied collisions between a sphere and a roller in the presence of an oil drop. They found that an increase in the viscosity of the oil leads to a decrease in the restitution coefficient, as is also predicted by the theory. Lian, Adams and Thornton [16] gave a closed-form solution for two spheres colliding in a liquid, in close agreement with the elastohydrodynamic theory [12].

More recently, Gondret *et al*. [17,18] observed the bouncing motion of spheres dropped onto flat surfaces immersed in a liquid. Joseph *et al*. [19] used a pendulum to perform collisions of swinging spheres with a vertical wall, again fully immersed in a liquid. These publications affirm the presence of a critical Stokes number, below which no rebound is seen. For larger Stokes numbers, the trends in the restitution data are similar to those predicted by the original elastohydrodynamic theory [12].

Davis, Rager and Good [20] observed the restitution behavior when spheres were dropped through air onto wet surfaces at impact velocities of approximately 1 m/s–5 m/s. They gave a closed-form expression for the wet restitution coefficient:

$$\begin{aligned} e &= e_{\mathrm{dry}}(1 - St_{\mathrm{c}}/St) && \text{for } St > St_{\mathrm{c}}, \\ e &= 0 && \text{for } St \le St_{\mathrm{c}}, \end{aligned} \tag{4}$$

where e is the ratio of the rebound speed as the particle exits the liquid layer to the impact speed as the particle enters the liquid layer on the solid surface, e_{dry} is the coefficient of restitution for a dry collision, and St_{c} is the critical Stokes number below which no rebound is seen for wet collisions. Their experimental data are in good agreement with Eq. (4), though with considerable scatter that is thought to be due to surface roughness affecting the final stages of impact. Recently, Kantak and Davis [21] showed that Eq. (4) can also describe the normal restitution coefficient (ratio of normal rebound velocity and normal approach velocity) in oblique impacts. They also developed order-of-magnitude estimates for predicting the loss of momentum in the tangential direction and the rotation imparted to a sphere undergoing an oblique collision with a wet wall. In related work, Joseph and Hunt [22] examined oblique particle-wall collisions immersed in a liquid. They introduced a coefficient of sliding friction for predicting the tangential force in terms of the tangential velocity. Their model estimates the coefficient of friction as a function of fluid viscosity, which in turn changes with increasing pressure and temperature in the fluid gap.

The theoretical works of both Kantak and Davis [21] and Joseph and Hunt [22] assume a flat profile of the gap between the sphere and the wall in estimating the tangential fluid force. A more complete model should include the nonuniform dynamic deformation profile of the colliding solids, as well as properly account for cavitation and the finite thickness of the liquid layer. To meet these needs, the present work provides an extension of elastohydrodynamic theory [12] to oblique collisions, including the effects of cavitation, the thin liquid layer, and the deformation profile on the tangential motion. Such a model would be useful in large-scale simulations of a system of wet particles undergoing collisions. However, the model should ideally be in the form of closed-form expressions, so that it can be efficiently incorporated into large-scale simulations, and so an additional purpose of the present work is to provide such closed-form expressions. Section 2.1 outlines modification of the theory of Davis *et al*. [12] for the solution of the normal component of motion. In Section 2.2, the theoretical development for the tangential component of motion is given, followed by the method of numerical solution in Section 2.3. The results of the numerical solutions are presented in Section 3, and closed-form expressions to match the numerical solution are provided in Section 4.

2. Theoretical development

Fig. 1 is a schematic of a sphere undergoing an oblique collision with a surface covered with a thin liquid layer. Kantak and Davis [21] used scaling arguments and verified experimentally that, at least as a first approximation, the normal component of motion can be considered to be decoupled from the tangential component. We use this concept and solve first for the normal component of motion and then use these results to solve for the tangential component of motion during an oblique collision. The solution is presented in terms of a sphere impacting a plane, but at least the normal component is readily extended to the collision of two spheres [12].