Available online at www.sciencedirect.com

ScienceDirect

Procedia Chemistry 10 (2014) 151 – 157

Procedia Chemistry

XV International Scientific Conference "Chemistry and Chemical Engineering in XXI century" dedicated to Professor L.P. Kulyov

On the strength calculation of the rotating parts

V.M. Belyaev[a] *, A.A. Zurilin[b], S.O. Cherkasov[a]

[a] *National Research Tomsk Polytechnic University, Lenin avenue, 30, Tomsk, 634050, Russia*
[b] *Yakutniproalmaz Research and Design Institute, Lenin street, 39, Mirny, Republic of Sakha (Yakutia), 678170, Russia*

Abstract

The existing solutions of differential equations of equilibrium of an infinitesimal element of the rotating parts of an isotropic elastic solid known as the Navier equilibrium equations are considered. Examples of the flat disk calculation by solving the differential equilibrium equations by the sweep method and the finite element method in the modern program "Autodesk Simulation Multiphysics" are represented; paradoxical changes of radial and hoop stresses are revealed. An original method of derivation formulas based only on the principle of d'Alembert to calculate radial and hoop stresses in parts that operate under centrifugal (inertial) forces is proposed. The solution for rotating disks of any profile that corrects unnatural classical solutions is obtained. Analysis of the obtained new formulas for calculating stresses shows that it is necessary to reject the concept of "equal-strength disk" because of the inability to provide the equality of the hoop and radial stress in all sections of the disk. A new method of the optimum strength disk profile calculation, which requires a restriction of outer radius disk, is suggested. In designing of optimum strength rotating parts is recommended to limit outer disk radius of $0{,}8\sqrt{[\sigma]/(\rho \cdot \omega^2)}$ where $[\sigma]$ − the allowable stress, ρ − density of the disk material; ω − angular velocity of disk rotation.

Peer-review under responsibility of Tomsk Polytechnic University

Keywords: elasticity theory; hoop and radial stresses; rotating parts

1. Introduction

Various machines such as centrifuges and separators for division or filtration of suspensions and emulsions, gas and steam turbine engines, turbochargers, etc. incorporate in their construction the rotating parts of various shapes,

* Corresponding author: Vasiliy M. Belyaev Tel.: + 8-903-915-5488
E-mail address: belyaev_vm@tpu.ru

Peer-review under responsibility of Tomsk Polytechnic University
doi:10.1016/j.proche.2014.10.026

which operate under inertial (centrifugal) force and surface loads. Problem of stress calculation in a rotating thin disk was considered for the first time in 1850 by Maxwell JC in the report "On the equilibrium of elastic solids" [1]. He used the works of Hooke R., Navier C.L.M.H., Poisson S.D., Lame G., Clapeyron B.P.E. and Stokes G.G. Stokes work was especially marked:

"…his equations are identical with those of this paper, which are deduced from the two following assumptions.

In an element of an elastic solid, acted on by three pressures at right angles to one another, as long as the compressions do not pass the limits of perfect elasticity –

1st. The sum of the pressures, in three rectangular axes, is proportional to the sum of the compressions in those axes.

2nd. The difference of the pressures in two axes at right angles to one another, is proportional to the difference of the compressions in those axes".

As it is known from history of the theory of elasticity[2], and to this day[3], differential equations of equilibrium of an elastic body in the analysis of operation of rotating details are used by huge number of scientists. These equations are also included in textbooks and reference books.

For example, the rotating circular disk that side surfaces are formed by the rotation of curve z = ±*f*(*r*) about the same axis is considered in[4] (Fig. 1). Using the equations of elementary volume equilibrium, generalized Hooke's law, and neglecting the axial disk displacement, the differential equilibrium equation of second-order from the amount of disk element displacement (u) at a distance r from the axis of rotation was obtained:

$$\frac{d^2u}{dr^2}+\left(\frac{1}{r}+\frac{1}{z}\cdot\frac{dz}{dr}\right)\frac{du}{dr}+\frac{1}{r}\left(\frac{\nu}{z}\cdot\frac{dz}{dr}-\frac{1}{r}\right)u+\frac{1-\nu^2}{E}\rho\cdot\omega^2\cdot r=0 \quad (1)$$

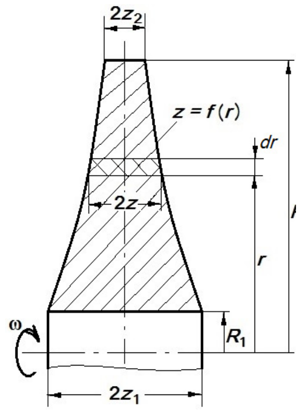

Fig. 1. Disk parameters

Analytical solution of this differential equilibrium equation can only be done for a flat disk (z = const). For a disk with a central hole and loads of p_1 and p_2, equation gives the formulas (2, 3) for calculating the hoop (σ_t) and radial (σ_r) stresses, where ρ − density of the disk material; ω - angular velocity of rotation of the disk; ν − Poisson's ratio; R_1, R_2 − hole radius and outer disk radius; r − radius of considered disk section.