Advanced Engineering Informatics 26 (2012) 603–617

Contents lists available at SciVerse ScienceDirect

Advanced Engineering Informatics

journal homepage: www.elsevier.com/locate/aei

Arc-elasticity and hierarchical exploration of the neighborhood of solutions in mechanical design

Arnaud Collignan [a], Patrick Sebastian [a,*], Jérôme Pailhes [b], Yann Ledoux [a]

[a] Université de Bordeaux, I2M-IMC, Esplanade des Arts & Métiers, 33405 Talence, Cedex, France
[b] Arts et Métiers Paristech, I2M-IMC, Esplanade des Arts & Métiers, 33405 Talence, Cedex, France

ARTICLE INFO

Article history:
Received 9 September 2010
Received in revised form 13 February 2012
Accepted 8 April 2012
Available online 5 May 2012

Keywords:
Optimization
Mechanical design
Arc-elasticity
Design space
Genetic algorithm

ABSTRACT

In most industrial design processes, the approaches used to obtain a design solution that best fits the specification requirements result in many iterations of the "trial-and-error" type, starting from an initial solution. In this paper, a method is proposed to formalize the decision process in order to automate it, and to provide optimal design solutions. Two types of knowledge are formalized. The first expresses the satisfaction of design objectives, relating to physical behaviors of candidate design solutions. This formalization uses three models, an observation one, an interpretation one and an aggregation one; every design solution is qualified through a single performance variable (a single objective function). The second model is related to modifications that may or may not be applicable to the pre-existing solution. The Designer is often able to define preferences concerning design variables. Some modifications related to this pre-existing solution, can be preferred to other ones. A hierarchy of design variables is proposed to formalize these preferences. The concept of arc-elasticity is introduced as a post-processing indicator to qualify candidate solutions through a trade-off between the performance improvement and their relative distances to the initial solution. The proposed method is used and applied to a riveted assembly, and a genetic algorithm is used to identify optimal solutions.

1. Introduction

In industrial processes, a typical occurrence in sub-contracting mechanical design, design activity is based on companies' know-how as well as on designer's imprecise knowledge [1]. These processes require several iterations between product design and simulation, starting from a predefined solution. The aim of this "trial-and-error" approach [2,3] is to obtain a product that corresponds to criteria defined in the design specification documents, via an iterative process of decision-making and optimization. Optimization is mostly based on knowledge rather than on numerical optimization methods. This time-consuming process provides no guarantee of approaching an optimal solution and no justification of the decision process. During such processes, designer preferences linked to the initial solution and to the behavior of the product are applied.

In the following paper, reasoning is based on a single solution, called the reference solution or initial solution since it is involved in an iterative process; this reference solution is related to designer preference and supports the mathematical formulation of this preference. Indeed, design activity is often underpinned by one pre-existing solution whose structure is regarded as preferable even if it is not precisely adapted to the ongoing design problem. The design and development (prototyping, testing, and industrialization) of this reference solution may require significant investment (costs and delays); and by the end of the process this solution is regarded as well known and secure. The structure of new design solutions is implicitly constrained to remain close to this initial solution as any difference between a candidate solution and the initial solution will imply additional costs [4]. Thus, it is possible to formalize user preferences as a distance between optimal and initial solutions.

The majority of product design optimization problems are regarded as being "multi-objective" [1]: satisfying one of the product's performance criteria, which are related to physical observation variables, is linked to the performance of the other observation variables. Ullman proposes a list of the main elements that must be taken into account in making decisions for this kind of problem: design alternatives and human preferences [5]. Human preference is therefore a major element in design and Augusto et al. [6] distinguishes *a priori* and *a posteriori* decision assistance to express these preferences. *A priori* decision support and designer's preferences concerning the product are formulated in the mathematical optimization problem in the same ways as the physical behavior model of the product.

* Corresponding author. Tel.: +33 (0) 556 845 414; fax: +33 (0) 556 845 436.
E-mail address: patrick.sebastian@trefle.u-bordeaux.fr (P. Sebastian).

1474-0346/$ - see front matter
http://dx.doi.org/10.1016/j.aei.2012.04.001

Numerous methods can be used to tackle multi-objective problems taking designer's preferences into account. Jones et al. [7] describes three of these methods, namely the reference point method, goal programming and compromise programming. Romero underlines similarities in these approaches. All of these methods are based on a precise modeling of the design objectives [8]. The optimization process then consists in minimizing the gap between the candidate design solutions and these objectives. Other methods use designer's preferences formalized through design decision-making methods [9]. Utility theory [10], introduced in economics, proposes to interpret an observation variable through a utility curve formalizing the preference. Marston illustrates this method [11]. Starting from the Utility theory, Antonsson has developed the Method of Imprecision (MoI) where multiple variables are qualified using fuzzy logic and are aggregated through design strategies [12,13]. These aggregation strategies are structured by design axioms to ensure their *design-ready* property [14]. The Observation-Interpretation-Aggregation method (OIA) is a similar process, structuring the modeling of product behavior and preferences through three models [15,16]. The first is a predictive model of the product's behavior; the second qualifies the observation variables using desirability curves [17], while the third aggregates the resulting interpretation variables, formalizing and prioritizing the design objectives *via* an aggregation step [18,19]. The OIA method is employed to quantify the product's performance through a single variable.

Later on, we consider two types of variables: the product's performance and the distance between one candidate solution and a reference solution. Then, the concept of arc-elasticity is introduced as a decision support indicator of the relative improvements or degradations in these two variables. Elasticity has been introduced in the area of microeconomics [20]; Allen and Lerner [21] proposes the principle of arc-elasticity based on the measurement of the elasticity between two points. In this paper, this concept is applied in engineering design to quantify trade-offs in the selection of different candidate solutions.

From this preference trade-off an original optimization method is proposed. The paper begins by defining the OIA modeling method and the three resulting models. In the second part, the arc-elasticity indicator is presented and adapted to the design field. Next, from this model and this indicator, we propose a global optimization method based on the sequential search for optimal solutions using a hierarchy of design variables. This process is described in the third part of the article. Finally, in the fourth part, the method is illustrated by applying it to a fastened assembly (with rivets) and by optimizing the mechanical system with a genetic algorithm.

2. Modeling methodology of performance

Multi-objective optimization problems consist in finding optimal values for every observation variable. Designer's expectations relative to these variables are generally conflicting [19]. Solving multi-objective optimization problems must be performed through a trade-off between the different candidate solutions. This trade-off, derived from an aggregation process, is used to pass from several variables to a single one.

We argue that designer's preferences deriving from his know-how and experience are not formalized in the observation model. However, designer's preferences are used to interpret every observation variable. Designers have information that enables them to decide whether a value is acceptable or not, information which can be formalized. In the following, preferences are formalized using mathematical functions. The Observation-Interpretation-Aggregation approach (OIA) is used to translate the design problem into a mono-objective function including both physical behavior and decision models. The OIA approach, detailed in the present section, is divided into three models (Fig. 1):

1. The observation model μ is a model of the behavior of the product (physics, economics, etc.).
2. The interpretation model δ expresses the design criteria translating physical observation variables into desirability levels.
3. The aggregation model ξ formalizes and defines priorities between the design objectives.

2.1. Design variables and observation model

Design variables (also called decision variables or design parameters according to some authors) are related to the main structural characteristics of the system that must be quantified by the designers and correspond to the degrees of freedom on which designers act to define the system; system performances result from the values of these variables. In the following, X is defined as the vector containing every design variable x_i. The design search space $\overline{\Omega}$ (Fig. 1) is defined as the space containing every candidate solution of the optimization problem. Therefore, this space is formed from every possible instantiation of the vector X.

Every design variable x_i is associated with a value domain bounding the admissible values of the variable. The value domain of x_i is denoted as $[\bar{x}_i^-;\bar{x}_i^+]$. Therefore the design search space and the design variables vector satisfy:

$$\begin{aligned} &X \in \overline{\Omega} \\ &\text{with } X = [x_1 \quad \ldots \quad x_i \quad \ldots \quad x_n]^T \\ &\text{and } \overline{\Omega} = \left\{ [\bar{x}_1^-;\bar{x}_1^+] \cdots [\bar{x}_i^-;\bar{x}_i^+] \cdots [\bar{x}_n^-;\bar{x}_n^+] \right\}^T \end{aligned} \tag{1}$$

Value domains can be continuous (dimensions, energy quantities, etc.) or discrete (materials, numbers of parts, product architectures, etc.). Their boundaries are defined by designers, from design requirement documents and also from his expertise.

An instantiated vector X defines a candidate solution. From this solution, the observation model computes the observation variables y_i, which forms the vector Y. These variables are used to observe the product's behavior from which the performances of the product are derived. The observation model verifies:

$$\mu(X) = Y \text{ with } Y = [y_1 \quad \ldots \quad y_i \quad \ldots \quad y_m]^T \tag{2}$$

2.2. Interpretation model

Observation variables are then translated into an interpretation variable (vector Z) through the interpretation model. We propose to build formal interpretation functions using desirability functions, but other "value functions" can be employed [22]. The desirability notion was first suggested by Harrington [17] in the area of quality, and is commonly used in multi-objective optimization

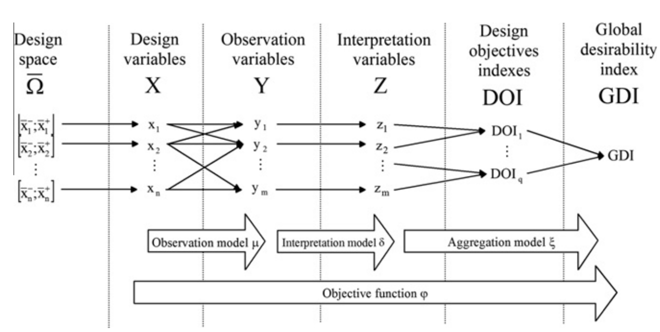

Fig. 1. Modeling methodology.