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Layout optimization for maximizing wind farm power production using sequential convex programming

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HIGHLIGHTS

- The continuous wake model describes well the wake profile behind a wind turbine.
- The expected wind farm power is expressed as a differentiable function.
- SCP can be employed to efficiently optimize the layout of a large-scale wind farm.
- The optimized wind farm layout increases the wind farm power efficiency by 7.3%.

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ABSTRACT

This paper describes an efficient method for optimizing the placement of wind turbines to maximize the expected wind farm power. In a wind farm, the energy production of the downstream wind turbines decreases due to reduced wind speed and increased level of turbulence caused by the wakes formed by the upstream wind turbines. As a result, the wake interference among wind turbines lower the overall power efficiency of the wind farm. To improve the overall efficiency of a wind farm, researchers have studied the wind farm layout optimization problem to find the placement locations of wind turbines that maximize the expected wind farm power. Most studies on wind farm layout optimization employ heuristic search-based optimization algorithms. In spite of their simplicity, optimization algorithms based on heuristic search are computationally expensive and have limitation in optimizing the locations of a large number of wind turbines since the computational time for the search tends to increase exponentially with increasing number of wind turbines. This study employs a mathematical optimization scheme to efficiently and effectively optimize the locations of a large number of wind turbines with respect to maximizing the wind farm power production. To formulate the mathematical optimization problem, we derive a continuous wake model and express the expected wind farm power as a continuous and smooth function in terms of the locations of the wind turbines. The constructed wind farm power function is then maximized using sequential convex programming (SCP) for the nonlinear mathematical problem. We show how SCP can be used to evaluate the efficiency of an existing wind farm and to optimize a wind farm layout consisting of 80 wind turbines.

1. Introduction

Among the various renewable energy sources, wind power has proven effective for large-scale energy production. However, for a large-scale wind farm, wind power production efficiency can deteriorate due to wake interference among the wind turbines. Wakes formed by the upstream wind turbines affect the wind speed behind the rotors and decrease the energy production of the downstream wind turbines due to reduced wind speed and increased level of turbulence. Levels of wake interference can differ depending on the relative locations of the wind turbines and wind direction. In this paper, we discuss a method for improving the expected wind farm power production by strategically placing the wind turbines in a wind farm site. Specifically, we construct the expected wind farm power function using a continuous wake model to capture the realistic wake flow. The expected wind farm power function is then maximized by applying a mathematical optimization algorithm.

The wind farm layout problem was first formulated by Mosetti et al. [1] to maximize the expected wind farm power (objective function) in terms of the locations (optimization variable) of wind

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turbines. The objective function is often expressed in terms of the expected wind farm power (or annual expected power) and economic costs, such as the costs of wind turbine, construction, electrical infrastructure, and land. For example, Mosetti et al. [1] studied the optimization of expected wind farm power taking into account the cost of the wind farm. Ozturk and Norman [2] examined the optimal net profit, which is defined as the difference between the electricity production earning and the installation cost of a wind farm. Mora et al. [3] and González et al. [4] dealt with optimizing the net present value of a wind farm taking into consideration the annual cash flow and the monetary discount rate. Various types of constraints have also been proposed to make the wind farm layout problem more realistic. These constraints include allowable wind turbine area, minimum wind turbine-inter distance, maximum number of wind turbines, maximum length for the electrical cables, etc. [5].

The most essential component of the objective function for the wind farm layout problem is the expected wind farm power function, which maps the relative locations of wind turbines to the averaged wind farm power production while taking into account the probability distribution of wind conditions. The wind farm power function is constructed based on a wake model, which is an analytical expression describing the wind speed behind a wind turbine. Wake is often described by its domain (shape) and value (in terms of ratio of wind speed reduction). For example, the Park wake model, which is the most widely used model, assumes that the wake expands linearly as it propagates in the downstream direction and that the wind speed inside the wake (3D expansion cone) is constant [6]. Based on the Park wake model, Katic et al. [7] constructed a wind farm power function by aggregating the influences of the multiple wakes by the upstream wind turbines on a downstream wind turbine. The constructed wind farm power function has been incorporated in many software, such as WAsP [8], WindFarmer [9], WindPRO [10] and OpenWind [11], to evaluate the wind farm energy production. The function has also been used extensively to construct the objective function for the wind farm layout problem. However, the use of a piece-wise linear wind speed profile for the Park wake model results in a non-smooth (non-differentiable) wind farm power function, which renders optimizing the wind farm power function using gradient-based optimization algorithms difficult.

According to González et al. [5], most studies on the wind farm layout problem explore the wind farm power function that is based on the Park wake model [6,7]. Due to the non-differentiability of this wind farm power function, heuristic optimization algorithms are often employed to solve the layout problem. Genetic Algorithm (GA) is one method that has been constantly employed to the wind farm layout problems [1,12–17]. Other population-based heuristic search methods, such as Monte Carlo simulation [18] and Simulated Annealing [19,20] have also been applied to the problem. Generally speaking, heuristic search methods attempt to iteratively improve a solution by sampling new solutions (exploration) over a discretized input space. The level of exploration is determined by some criteria designed to measure the improvement of the objective function. Because the possible solutions are searched over a discretized space, the efficiency of heuristic-based algorithms strongly depends on the size of discretization. A fine discretization for the search space would improve the effectiveness on finding an optimal solution, but would also incur high computational costs.

To overcome the computational cost incurred due to discretized input space, Kusiak et al. [21] employed an Evolutionary Algorithm (EA) for the optimal layout problem by treating the farm area as a continuous space. Other approaches such as Ant Colony Optimization (ACO) [22], Particle Filtering method [23], Extended Pattern Search (EPS) [24], and Particle Swamp Optimization (PSO) [25–28] have been proposed to search a good solution over a continuous domain. Wan et al. [25] and Chowdhury et al. [28] have shown that the PSO algorithm can efficiently optimize 36 and 25 wind turbines, respectively, with reasonable computational time (i.e., within 2000 iterations). Recently, Wilson et al. [29] has deployed a developmental model-based algorithm, called DEVO-II, that mimics the gene regulatory network and efficiently optimizes around 400 wind turbines with less than 100 function evaluations.

Although there have been many studies on wind farm layout problem, little progress has been made in understanding the wake interference among wind turbines and to construct more realistic wind farm power functions. Wind tunnel tests [30,31] and computational fluid dynamics simulations [32] have been used to study the wake interference phenomenon between the wind turbines. These studies have shown that a wind speed profile inside a wake resembles an inverted Gaussian shape, with the wind speed varying as a continuous function in both the downstream and the radial directions, in contrast to the Park wake model that assumes a piece-wise linear wind speed profile. Incorporating these recent developments, we derive a continuous and smooth wind speed profile that allows the wind farm power function to be expressed as a smooth (differentiable) function of wind turbine location variables. We then formulate the wind farm layout problem as a nonlinear mathematical optimization problem and solve it using sequential convex programming (SCP). In this framework, various additional cost functions as well as constraints can be included flexibly. Furthermore, the wind farm layout problem can be efficiently solved by exploiting the gradient and Hessian of the objective function, even for a wind farm with a large number of wind turbines.

This paper is organized as follows: first, we derive the wind farm power function constructed using a continuous wake model. The derived wind farm power function is then calibrated using CFD simulation data. The wind farm layout optimization problem is then formulated and solved by applying sequential convex programming. The proposed optimization approach is used to evaluate the efficiency of an operating wind farm and to study the optimal layout of a wind farm. The paper is concluded with a brief summary and discussion.

2. Derivation of wind farm power function

This section describes the derivation of a wind farm power function to be used as the objective function for the wind farm layout optimization problem. The wind farm power function is derived as follows: The power expression for a single wind turbine, under undisturbed wind flow condition, is described based on the actuator disc model in aerodynamics. A continuous wake model to represent the wind speed profile behind a wind turbine is then discussed. The power of a downstream wind turbine under the influence of *a single wake* formed by a single upstream wind turbine is derived based on momentum conservation theory. The power of a downstream wind turbine under the influence of *multiple wakes* caused by upstream wind turbines is constructed by aggregating the energy deficiencies due to wakes. Finally, the wind farm power function is expressed as the sum of the powers of all the wind turbines in a wind farm.

2.1. Power function of a wind turbine

The power of a single wind turbine due to a wind flow with wind speed U can be obtained based on the actuator disc model in aerodynamics [33]. According to the actuator disc model, the wind turbine power can be quantified by the amount of power extracted