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Comparing hyperbolic, delay-amount sensitivity and present-bias models of delay discounting

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ABSTRACT

Delay discounting is a widely studied phenomenon due to its ubiquity in psychopathological disorders. Several methods are well established to quantify the extent to which a delayed commodity is devalued as a function of the delay to its receipt. The most frequently used method is to fit a hyperbolic function and use an index of the gradient of the function, *k*, or to calculate the area under the discounting curve. The manuscript examines the behavior of these quantification indices for three different datasets, as well as provides information about potential limitations in their use. The primary limitation examined is the lack of mechanistic specificity provided by either method. Alternative formulations that are thought to provide some mechanistic information are examined for the three separate datasets: two variants of a hyperboloid model (Rachlin, 1989, Judgment, decision and choice. New York: W.H. Freeman) and the quasi-hyperbolic model (Laibson, 1997, Q. J. Econ., 112, 443–477). Examination of the parameters of each formulation suggests that the parameters derived from the quasi-hyperbolic model allows groups and conditions within the three datasets to be reliably distinguished more readily than the hyperboloid models. However, use of the quasi-hyperbolic model is complex and its limitations might offset its ability to discriminate within the datasets.

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1. Introduction

Delay or temporal discounting is a process by which individuals derive the subjective value of a commodity, which is available following a delay, by creating a composite of the commodity's magnitude and the delay to its receipt (e.g., Peters and Buchel, 2011). Researchers are interested in this process for several reasons. First, from a basic science perspective, calculations of this type are common in everyday life and understanding the neural and psychological bases for these calculations provides information about critical processes driving decision-making. Second, the value of delayed commodities is lower in individuals exhibiting a wide variety of psychopathologies, including substance dependence, pathological gambling, attention deficit hyperactivity disorder and conduct disorder (e.g., Bickel et al., 2012; Perry and Carroll, 2008; Robbins et al., 2012; see Weafer et al., 2014 for a recent review); implying that the delay discounting process is quantitatively, or possibly qualitatively, different in diagnosed individuals compared to undiagnosed individuals.

Studies examining delay discounting in human participants examine choices between smaller, sooner and larger, later rewards. Obtaining preferences between the two types of reward permits researchers to identify the value of the smaller, sooner reward that is equivalent to the larger, later reward over a series of specific delay values ("indifference points" at the specific delays). There are numerous tasks to do this, many of which are described and critically evaluated by Madden and Johnson (2010). When indifference points have been identified for a series of delays, researchers quantify the subjective value for the larger reward as a function of the delay in two main ways.

One way is to fit mathematical models to the data. The most widely-used model is based on a hyperbolic function:

$$V = \frac{A}{1 + kD} \tag{1}$$

where V represents the subjective value of the larger, later reinforcer (indifference point), A represents the magnitude/amount of the larger, later reward, k is a fitted parameter that measures

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steepness of the discounting curve with larger values indicating greater/steeper discounting, and *D* represents the delay to the delivery of the larger, later reward (Mazur, 1987; see Killeen, 2009 for a discussion of whether subjective utility provides a better metric than subjective value). While this model has been used extensively, as with any fitting procedure, there are questions relating to what is the appropriate metric to quantify goodness of fit for this nonlinear model, the threshold level of the different metrics to determine whether a fit is "acceptable", and what should be done when there are systematic residuals to the fit, which suggest that the function does not capture the underlying discounting process adequately (Johnson and Bickel, 2008).

A second way to quantify delay discounting that has been widely adopted is to assess the area under the "curve" (AUC) created by plotting the indifference points at each delay. That is, the AUC is calculated by summing the areas of the normalized trapezoids formed by consecutive indifference points at each delay value (Myerson et al., 2001). Due to normalization, an AUC of 1.0 is associated with indifference points that are both equal to the objective value of the larger, later reward and invariant as a function of delay. As the AUC decreases toward a minimum of 0.0, the effect of delay on the indifference points at each delay, which represent the subjective values, is expected to be more pronounced. The AUC method does not assume any specific mathematical form and so goodness-of-fit issues are not a concern. However, like any AUC function, very different patterns of indifference points may produce identical summary AUC values, making it difficult to draw conclusions about the effects of delay on subjective value based solely on AUC information.

Both of these quantification methods enable researchers to describe the effects of the delay discounting process under various conditions and as a result of various manipulations (e.g., Green and Myerson, 2004, 2013; Odum, 2011). However, neither provides information about the mechanisms by which variables, such as the characteristics of the participants or size of the larger, later reward, affect the degree of discounting (Bickel et al., 2014; Mackillop, 2013). In this manuscript, we use three datasets to explore the behavior of *k*, the slope of the hyperbolic function, and AUC between groups and across conditions. These datasets are then used to look at other commonly used quantification procedures that have been proposed to provide additional information about factors underlying the delay discounting process.

2. Description of datasets used

Two datasets focus on comparisons between different groups of individuals (the ADHD and the SMOKING datasets) and one dataset focuses on comparisons between two delayed reward amount conditions collected using a within subject design (AMOUNT dataset). The delay discounting task used in all datasets was based on that described in Mitchell (1999), though delays and amounts differed between datasets (Table 1).

All discounting data were assessed for systematicity (Johnson and Bickel, 2008: Criterion 1); that is, beginning with the second shortest delay, an indifference point was judged to be nonsystematic if it was larger than the indifference point for the preceding delay by more than 20% of the larger, later reward. Participants with one or more nonsystematic indifference points were excluded from all analyses that we reported. In the ADHD dataset, 39 participants were excluded (22 ADHD-diagnosed and 17 undiagnosed) from a total of 240 initial participants. In the SMOKING dataset, all data was systematic. In the AMOUNT dataset, 2 participants generated nonsystematic data on $10 task and 1 was nonsystematic on the $100 task; data from these three individuals were consequently excluded from all $10 and $100 task data analyses. All equation fits were performed using the Excel 2010 Solver add-in (Microsoft, Redmond WA).

The ADHD dataset examined here includes 105 ADHD-diagnosed and 96 undiagnosed children, aged 9.28 and 8.70 years (SD = 1.28 and 1.07). Some of the data were previously published in Wilson et al. (2011), but additional data have been added from individuals recruited as part of the continued research efforts. All participants were recruited in the same way as described in Wilson et al. (2011), that is, a two-stage process was used to generate information that could be presented to a clinical diagnostic team. Each team member arrived at a 'best estimate' diagnosis for ADHD independently using DSM-IV TR criteria (American Psychiatric Association, 2000 [APA]). If consensus was not readily achieved, the child was excluded from the study. To be assigned a diagnosis of ADHD, the following conditions had to be met: (1) the child's symptoms could not be better accounted for by another disorder, (2) evidence of impairment had to be apparent, e.g., high impairment ratings on the Strengths and Difficulties Questionnaire (Goodman, 2001), in the parent/teacher comments, or in the school record, and (3) a cross-situational presentation was required, i.e., some elevation in both parent and teacher reports. Exclusion criteria included current major depression or learning disability, or history of mania, psychosis, or autism spectrum disorder. ADHD-diagnosed children who were prescribed non-stimulant medication were excluded and those children prescribed stimulant medication underwent a 24–48-h washout prior to testing, dependent on the specific medication prescribed. ADHD-diagnosed and undiagnosed children were followed for three years and a complete diagnostic and neurocognitive assessment was completed at each annual assessment. Only data drawn from the first year for children whose diagnosis was consistent across the three annual assessments were included in the dataset.

The SMOKING dataset includes delay discounting data from 120 individuals (60 regular smokers and 60 never smokers, aged 31.25 and 30.45 years with SD = 9.42 and 9.36). The majority of data were drawn from Mitchell and Wilson (2012), Experiments 1 and 2, with additional data collected during another study (in preparation). Regular smokers reported that they had smoked an average of at least 15 cigarettes each day for the past year or longer, while never

Table 1
Comparison of the parameters of the delay discounting tasks used in the three datasets.

Dataset	Outcome type	SS delay (days)	SS amount	LL delay	LL amount
ADHD	Hypothetical	0	$0–$10.50	0, 7, 30, 90, 180 days	$10
SMOKING	Potentially real; hypothetical[a]	0	$0–$50	2, 4, 8, 14, 22 weeks	$50
AMOUNT	Hypothetical	0	$0–$10.50[b] $0–$105	0, 7, 30, 90, 180, 365 days	$10char_dot $100

SS: smaller, sooner reward alternative.
LL: larger, later reward alternative.

[a] ANOVAs revealed no differences between the indifference points collected for each of the five delays for the hypothetical reward delivery conditions and conditions in which rewards were potentially real, i.e., one question was selected at random and payment delivered according to the participant's preference for that question; this lack of difference is consistent with data reported in Madden et al. (2003, 2004),).

[b] The order in which the $10 and $100 tasks were administered was counterbalanced between participants.