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Extended ordinary differential equation models for solar heating systems with pipes

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HIGHLIGHTS

- Extended ordinary differential equation models are proposed for solar heating systems.
- The models take into account pipe effects.
- Validation and efficiency of the models are demonstrated based on measured data.
- The advantages of the models are discussed compared to conventional models.
- Generalizability and applicability of the models are discussed.

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ABSTRACT

There is no doubt on the importance to investigate and develop solar heating systems. Mathematical modeling is the theoretically established tool for this purpose. In this study, improved, validated ordinary differential equation (ODE) models (an extended linear and a nonlinear ones) are proposed for a wide sort of solar heating systems with a solar collector, a heat exchanger, a storage and pipes. The pipes are considered as separated components in the model, so their thermal and delaying effects are taken into account. The advantages of the proposed ODE models, compared to other ODE, partial differential equation and delay differential equation (DDE) approaches are discussed. Based on the comparison of measured and simulated data of a real solar heating system, the validation and the efficiency of the proposed ODE models are demonstrated.

The applicability and the generalizability of the models are also discussed along with future research proposals.

1. Introduction

There is no doubt on the importance to investigate and develop solar heating systems. Mathematical modeling is the most widely used and theoretically established tool for this purpose.

In this study, a wide sort of solar heating systems are investigated having a solar collector, a heat exchanger and a storage as the main working components. The neighbouring components are connected with pipes, through which a pump circulates some heat transfer fluid (see Fig. 1).

In many works, the thermal (heat capacity, heat loss) and delaying effects of the pipes are neglected. This is the case, when solar heating systems are modeled with conventionally used ordinary differential equations (ODEs). In [1,2], collector-storage systems without a heat exchanger are modelled in such a way. In [2], the collector and the storage are divided into several layers, which are characterized with homogeneous temperatures forming a system of ODEs. In [3,4], collector-heat exchanger-storage systems are modelled with a linear ODE, the nonlinear version of which can be found in [5].

Pipe effects can easily become significant in systems with long pipes, for example, in case of district heating systems, or even in case of solar heating systems, when the collector field is relatively large, or when the collector field (eventually with a heat exchanger) can be installed only far from the swimming pool to be heated because of local circumstances (see e.g. [6,7]). Generally speaking, the case of solar power plants is a further example.

In [8], a partial differential equation (PDE) that is the linear transport equation corresponding to mass density is used in

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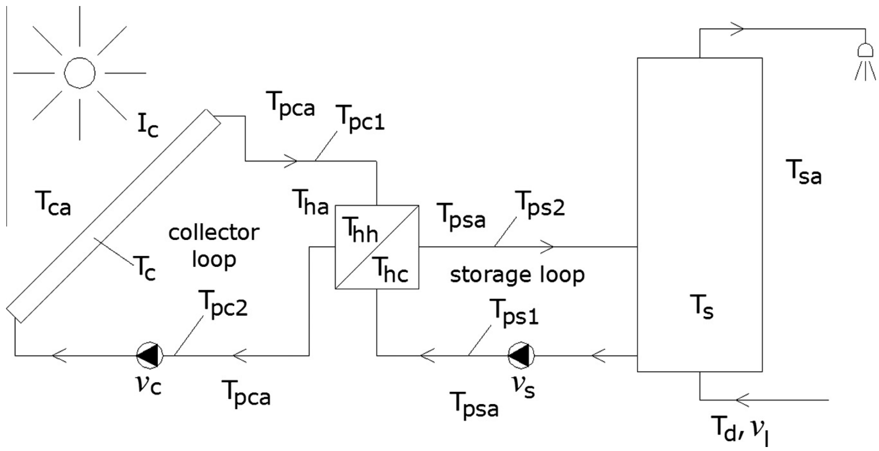

Fig. 1. Scheme of the investigated solar heating system.

modeling specific total energy of the fluid in the pipes of district heating systems. A further PDE resulted from this approach is solved along the characteristic path determined by the fluid particle motion. Then the pipes are discretized, the temperatures of which are calculated numerically. Another discrete approach can be found in [9], where an air conditioning system is considered as a dynamical system evolving in discrete time steps. The used discrete model is the so-called cellular automaton. The pipes of the system are composed of finite cells with equal fluid mass, based on which the outlet temperature is modelled.

The most frequently used model, with which the thermal and delaying effects of a pipe are considered, is the linear heat transfer PDE corresponding to plug-flow. Namely, the model is the linear transport equation corresponding to the pipe (fluid) temperature. This PDE pipe model is used to describe pipe temperature distribution inside the collector of a simple solar heating system in [10], inside single- and double-side heat exchangers along with connecting pipes under negligible heat losses in [11], inside an industrial chemical tubular continuous reactor in [12], inside the control volumes, into which a built-in-storage solar water heater is divided in [13], inside a flat-plate solar collector in [14] and inside a district heating system in [15]. In [16], the pipe wall temperature distribution is also described with a further PDE associated with the already mentioned PDE corresponding to the pipe fluid.

Some examples in literature work with delay equation models containing directly the time delay of a pipe for describing the temperature distribution or the outlet temperature as a function of time [17]. The delay approach is often more natural and convenient in handling such problems than the more conventional PDE and ODE approaches [18]. In [17,19], the heat transfer PDE is used to derive a delay equation (equation with time delay), which describes the pipe outlet temperature in time in case of constant and variable flow rates, respectively. These works contain relatively simple delay equations for a single pipe. The application of delay differential equations (DDEs) is the natural extension of the delay approach for complex heating systems. DDEs are special ODEs (or PDEs), which contain time delay, so former states affect the current states in such models. Most works on thermal engineering problems belong to the case of constant time delay. See e.g. [20] on the design of a control to regulate the heating of living tissue based on a nonlinear DDE, [21,22] on the DDE model of heat conduction in case of that there is a time delay of the heat flux vector and [23] on the DDE model of mixed problems for diffusion and reaction–diffusion equations. See [24] on a DDE model (of the so-called neutral type) for combustion systems. Some of the above works also serve with PDEs with delays. DDE models for the case of variable flow rate, and hence for variable time delay, are proposed in [18] based on the ODEs of [1,3], which serve also as the basis for the ODEs worked out in the present paper.

After the mentioned physically-based models, it is finally worth mentioning that heating systems can be also described with heuristic models, which consider the whole system (including pipe effects) as a black-box (without dividing it into a heater, pipes, etc.) and try to recognize rules on the output(s) as a function of the input(s). Such approaches can be found e.g. in [25,26], where artificial neural networks (collection of individually interconnected processing units) are used and in [26], where genetic algorithms (an optimum search technique) are applied to model solar heating systems.

In this study, improved ODEs for a wide sort of solar heating systems with a heat exchanger are worked out taking into account the pipe effects. The improved ODEs are modified and extended versions of the ODEs of [1,3,5], which do not deal with the pipe effects, nevertheless, they have been used successfully in a number of different applications (e.g. in [4,5,18,27]). The contributions of the present work in details are the following:

1. Improved linear and nonlinear ODE models considering thermal (heat capacity, heat loss) and delaying effects of pipes are proposed for modeling a wide sort of solar heating systems with a heat exchanger to fulfil the research gap concerning the lack of relatively simple ODE models taking into account pipe effects. The proposed ODEs are extended versions of already successfully used ODE models, which underlies their usefulness and applicability.
2. Based on measured data of a real system, the validation and the efficiency of the proposed ODEs are demonstrated, especially, compared to ODE and DDE models used in literature. It is also shown with comparison of simulated and measured results that the extended models are more reasonable to apply in practice than the basic models.

The paper is organized in the following way: Section 2 presents the investigated solar heating system type along with its basic linear and nonlinear ODE models. In Section 3, the improved, extended linear and nonlinear ODE models are introduced considering pipe effects. Section 4 contains the validation of the worked out ODEs for a real solar heating system based on comparison of measured and simulated data. It is also shown here that the extended models are more reasonable to apply in practice than the basic models. Final conclusions and future research proposals are given in Section 5.