J. Dairy Sci. 89:3968–3971

Technical Note: Estimating Statistical Power of Mixed Models Used in Dairy Nutrition Experiments[1]

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ABSTRACT

Statistical power is defined as the probability of *correctly* rejecting the null hypothesis. Power calculations may be useful in planning experiments. The objective of this technical note is to outline an applied method that estimates statistical power of a dairy nutrition experiment that employs a Latin square as the experimental design. Because the SAS MIXED procedure (PROC MIXED) is commonly used to analyze data sets, this note outlines basic programming procedures that may be used to estimate statistical power of a mixed model using this procedure.
Key words: statistical power, Latin square, mixed model, SAS

Dairy nutrition experiments are often designed to test a null hypothesis that dietary treatments have no effect on some dependent variable, often milk yield. In testing this null hypothesis (**H_O**) investigators risk making 1 of 2 types of errors. The first is to conclude that the dietary treatment has an effect when, in reality, it does not; this would be a type I error. The second error is to conclude that the dietary treatment has no effect when, in reality, it does; this would be a type II error. Investigators are often concerned with avoiding false claims in the form of type I errors; thus α, or the type I error rate, is usually set at 0.05.

Statistical power is defined as the probability of avoiding a type II error or, in other words, the probability of *correctly* rejecting the null hypothesis. If H_O is *true* then the associated F statistic, or F_O, has a central F distribution with 2 parameters, the numerator (v_1) and denominator (v_2) degrees of freedom. When H_O is *false*, F_O has the noncentral F distribution that depends upon v_1 and v_2. In this case, the noncentrality parameter (λ) may reflect the magnitude of the treatment effect or, in other words, just how wrong the traditional null hypothesis is (Murphy and Myors, 2004).

The use of SAS (SAS Institute Inc., Cary, NC) and mixed linear models (Littell et al., 1996) in the animal sciences has been described in several publications (Littell et al., 1999; Wang and Goonewardene, 2004). The SAS program may also be used to estimate power by referring to a noncentral F distribution and required parameters at a specified α level. The objective of this technical note is to outline an applied method that estimates statistical power of an experiment that employs a Latin square as the experimental design. Because the SAS MIXED procedure (PROC MIXED) is used to analyze data sets that include random effects or repeated measurements or both, this note will outline basic programming procedures that may be used to estimate statistical power of a mixed model using this procedure. For a detailed outline of linear model background, readers are referred to Stroup (1999).

Data Used in Illustration

Data from a nutrition study using 20 lactating Holstein cows are used to demonstrate statistical power estimation of a mixed model. In this experiment cows were randomly assigned to 1 of 5 Latin squares. During each of the 4 periods, cows were offered 1 of 4 diets, each an experimental treatment. Contrasts will be used to test for linear and quadratic trends in the milk production means.

Statistical Methodology

A Latin square design employs a 2-blocking scheme in which both cow and experimental feeding period facilitate comparisons of a third factor, namely dietary treatment (Tempelman, 2004). Individual squares are often replicated to increase the power of the statistical test. Cow nested within square is considered a random source of variation, whereas periods and treatments are considered to have fixed effects. Sometimes repeated measures, or multiple measurements made on the same cow during the same period, made sequentially over time, are also

Received November 11, 2005.
Accepted April 4, 2006.
[1]A contribution of the University of Nebraska Agricultural Research Division, Lincoln, NE 68583; Journal Series No. 15031.
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taken in these studies. The linear model as described by Kaps and Lamberson, (2004) is written as follows:

$$y_{ijk(m)} = \mu + \tau_m + \beta(\tau)_{im} + \rho(\tau)_{jm} + \alpha_{(k)} + \varepsilon_{ij(k)m}, \quad [1]$$

where $y_{ij(k)m}$ represents observation $ij(k)m$; μ represents the overall mean; τ_m represents the fixed effect of square m; $\beta(\tau)_{im}$ represents the random effect of cow i within square m; $\rho(\tau)_{jm}$ represents the fixed effect of period j within square m; and $\alpha_{(k)}$ represents the fixed effect of treatment k. The residual terms $\varepsilon_{ij(k)m}$ are assumed to be normally, independently, and identically distributed with variance σ_e^2.

Estimation of Power Using SAS

Stroup (1999) outlines how the MIXED procedure of SAS may be used to compute the noncentrality parameter, and this is used to determine power. This approach can be broken down into 3 parts:

1. In step 1 a new data set "new" is created from the original "milk." The data set "milk" has the design structure of the replicated (5 times) Latin square used in the experiment, which is listed in the Appendix of this paper. The "milk" data set lists the square and treatment allocation of each of the 20 cows during each of the 4 periods. It is important that the new data set has the same structure. If this is not done the PROC MIXED step will not compute the correct noncentral F parameter using the correct degrees of freedom in assessing power. The observed or proposed means (μ_i) for each corresponding treatment can be assigned. In our example treatment means of 33.4, 33.8, 33.6, and 36.6 were observed for treatment 1 through 4, respectively.

```
DATA new;
SET milk;
IF trt = 1 THEN mu =33.4;
IF trt = 2 THEN mu =33.8;
IF trt = 3 THEN mu =33.6;
IF trt = 4 THEN mu =36.6;
RUN;
```

2. In step 2 the covariance parameter estimates are defined in the PARMS statement. The estimate of σ_e^2, the error variance component, is listed (28.86). In addition, the variance among experimental units within square is defined as a RANDOM effect, and σ_c^2 (37.37) is also listed. If the investigator is planning an experiment based on the observations of a similar one conducted and analyzed, these estimates can be found in the original SAS output in the listing of *Covariance Parameter Estimates*. The NOPROFILE and NOITER options are used to set the variance components. The MODEL and CONTRAST statements are used to compute the F values for each test of interest. Then PROC MIXED is run using these estimates. In summary, this step reflects the design of the experimental layout, pattern of means specified, and the magnitude of the variance among experimental units given in the PARMS statement. Lastly, the ODS statement is used to create the data set b and an additional data set c for computations needed in the contrast statement. If the experimental design includes repeated measures of the response variable, the REPEATED statement can be included after the RANDOM statement, and the corresponding variances and covariances are listed as additional parameters in the PARMS statement.

```
TITLE 'Power of test: Milk Yield';
PROC MIXED noprofile DATA=new;
CLASS sq per cow trt ;
MODEL mu=trt per(sq) sq;
RANDOM cow(sq);
PARMS (37.3720) (28.8617)/NOITER;
CONTRAST 'Linear' trt -3 -1 1 3;
CONTRAST 'Quadratic' trt 1 -1 -1 1;
ODS OUTPUT tests3=b contrasts=c;
RUN;
```

3. In step 3 data contained in data sets b and c are used to compute power at the specified α level, 0.05. The FINV function returns the F value for the specified parameters v_1, v_2, and type I error rate (α). Both FINV and PROBF are SAS functions that are used to calculate the critical F value and the β value associated with the Type II error rate. The PROBF returns the probability of retaining a false null hypothesis, and subtracting this from 1 yields the

Table 1. Resulting output of the SAS program of statistical power denoting power of the test among treatment means and for linear and quadratic contrasts

Obs	Effect	Num DF	Den DF	FValue	ProbF	Label	alpha	fcrit	power
1	TRT	3	42	1.58	0.2090		0.05	2.82705	0.38479
2	PER(SQ)	15	42	0.00	1.0000		0.05	1.91175	0.05000
3	SQ	4	15	0.00	1.0000		0.05	3.05557	0.05000
4		1	42	3.06	0.0875	Linear	0.05	4.07265	0.40131
5		1	42	1.17	0.2853	Quadratic	0.05	4.07265	0.18469