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Performance comparison of six numerical methods in estimating Weibull parameters for wind energy application

Tian Pau Chang *

Department of Computer Science and Information Engineering, Nankai University of Technology, Nantou 542, Taiwan

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ABSTRACT

Two-parameter Weibull function has been widely applied to evaluate wind energy potential. In this paper, six kinds of numerical methods commonly used for estimating Weibull parameters are reviewed; i.e. the moment, empirical, graphical, maximum likelihood, modified maximum likelihood and energy pattern factor method. Their performance is compared through Monte Carlo simulation and analysis of actual wind speed according to the criterions such as Kolmogorov–Smirnov test, parameter error, root mean square error, and wind energy error. The results show that, in simulation test of random variables, the graphical method's performance in estimating Weibull parameters is the worst one, followed by the empirical and energy pattern factor methods, if data number is smaller. The performance for all the six methods is improved while data number becomes larger; the graphical method is even better than the empirical and energy pattern factor methods. The maximum likelihood, modified maximum likelihood and moment methods present relatively more excellent ability throughout the simulation tests. From analysis of actual data, it is found that if wind speed distribution matches well with Weibull function, the six methods are applicable; but if not, the maximum likelihood method performs best followed by the modified maximum likelihood and moment methods, based on double checks including potential energy and cumulative distribution function.

1. Introduction

Two-parameter Weibull distribution function has been commonly used in many fields including wind energy assessment, rainfall and water level prediction, sky clearness index classification, life length analysis of material, etc. Recently it even became a reference distribution in commercial wind energy software such as Wind Atlas Analysis and Application Program [1]. Wind power is proportional to the cube of wind speed, estimating the speed distribution for a particular wind farm is very important. Weibull scale parameter controls the abscissa scale of a plot of data distribution. Shape parameter describes the width of data distribution, the larger the shape parameter the narrower the distribution and the higher its peak value.

For a given data set several numerical methods can be applied to estimate the Weibull parameters. For example, the widely used moment method, empirical method, graphical method, maximum likelihood method, modified maximum likelihood method and energy pattern factor method [2–11], these will be revisited in latter section. Seguro and Lambert [10] concluded that the maximum likelihood method performs better than the popularly used graphical method in determining Weibull parameters; but the graphical method's performance can be enhanced as the bin size of wind speed is reduced. Akdag and Dinler [11] reviewed three conventional methods, i.e. the graphical, maximum likelihood and moment methods and proposed a new method (called energy pattern factor method) for estimating Weibull parameters. They stated that the new method has better suitability than others based on the comparisons of power density and mean wind speed. Jowder [12] used the empirical and graphical methods to analyze the wind power density at 10, 30, and 60 m heights in Kingdom of Bahrain, two Weibull parameters were estimated and compared. It is found that the empirical method provides more accurate prediction of average wind speed and power density than the graphical method. Sulaiman et al. [13] analyzed the wind data in Oman, the Kolmogorov–Smirnov test was adopted to examine the goodness of a Weibull function in fitting to observation data considering 1% and 5% of significant level. Dorvlo [14] analyzed the wind data from four stations in Oman and concluded that the Chi-square method gave better estimations for Weibull parameters than the moment and graphical methods, based on the Kolmogorov–Smirnov statistic. All the numerical methods mentioned are based on the fact that wind speed data follow the Weibull probability distribution. However the wind data actually observed is not necessary with the Weibull distribution. To analyze the accuracy of the

* Fax: +886 49 2561408.
E-mail address: t118@nkut.edu.tw

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Nomenclature

B	best bin size of histogram, m/s
c	scale parameter of Weibull function, m/s
E_a	wind energy per unit area by time-series data, kW h/m^2
E_{pf}	energy pattern factor, dimensionless
E_w	wind energy per unit area by Weibull function, kW h/m^2
$f(v)$	Weibull pdf
$F(v)$	cumulative Weibull function
k	shape parameter of Weibull function, dimensionless
n	sample size, dimensionless
$O(v)$	cumulative frequency of time-series data
Q	maximum error of cdf
R_n	random number within [0, 1]
T	time period, h
v	wind speed, m/s
$\bar{v}$	mean wind speed, m/s
$\overline{v^3}$	mean of wind speed cubes, m^3/s^3
v_i	observed wind speed in stage i, m/s

Greek letters

σ	standard deviation of wind speed, m/s
$\Gamma()$	Gamma function
ρ	air density, kg/m^3

numerical methods, an objective way that can be manipulated easily is the Monte Carlo simulation. Ghosh [15] developed a FORTRAN program to generate random samples following a specified Weibull distribution. Two Weibull parameters are estimated using both the graphical and maximum likelihood methods. Genc et al. [16] as well as Kantar and Senoglu [17] had compared some numerical techniques in terms of accuracy in calculating Weibull parameters through Monte Carlo simulations. However the scale parameters they used are all below 1.5 m/s, which are quite likely lower than the cut-in speed for most of wind generators currently used. As known, mean wind speed is about 10% lower than the scale parameter if its shape parameter is 2 around. On the other hand, sample sizes they used are about one hundred, which are far less than the hours of a month (720) not really applicable in yearly or long-term wind energy assessment.

The evaluation of Weibull parameters is so crucial in wind energy application. However, the accuracy of the six methods mentioned has rarely been discussed completely so far in a single research. In this paper, their performance will be investigated through Monte Carlo simulation considering some appropriate Weibull parameters and sample sizes, and analysis of actual observation data. Random variables having Weibull distribution are generated by computer system with specified shape and scale parameters. Wind speed data used are observed per 10 min from 2006 to 2007 at three wind farms experiencing different weather conditions in Taiwan. The first station Dayuan is located at the northwestern plain of Taiwan having strong wind in winter months, the height of anemometer is 64.7 m above ground level. The second station Hengchun is at the southern peninsula experiencing more stable weather conditions throughout the year, with the same anemometer height as Dayuan. The third station Penghu is at a small island in Taiwan Strait experiencing the highest wind in winter and spring, anemometer height is 46 m above ground level. Wind speed is transferred to hourly data in subsequent calculations.

2. Methods for evaluating Weibull parameters

Weibull distribution can be described by its probability density function $f(v)$ and cumulative distribution function $F(v)$ given as:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{1}$$

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{2}$$

where v is the wind speed, k the dimensionless shape parameter, and c is the scale parameter having the same unit with v. The distribution is named Rayleigh distribution if its shape parameter k is 2.

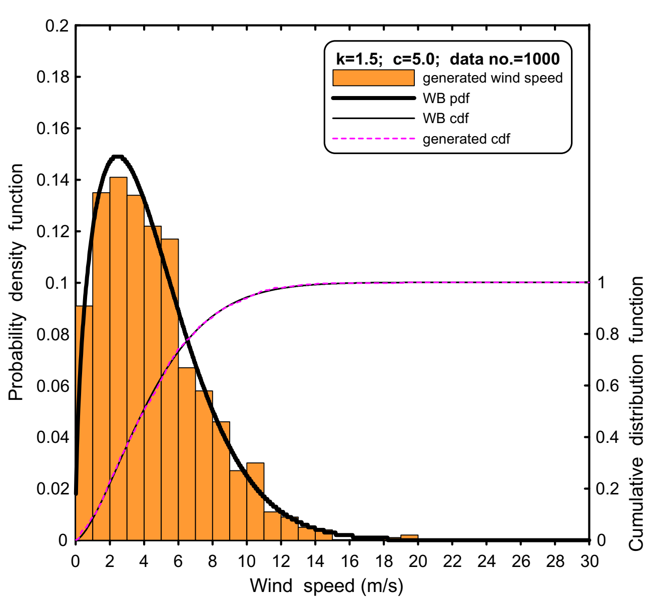

Fig. 1. Histogram of 1000 random variables generated with $k = 1.5$ and $c = 5$.

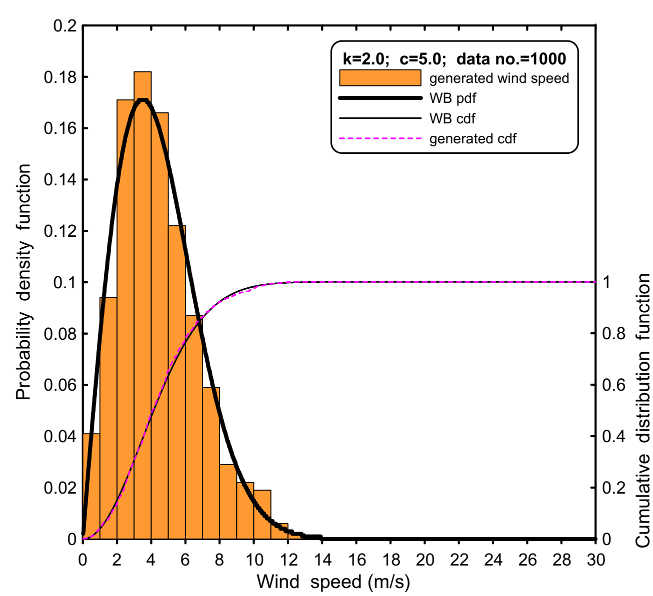

Fig. 2. Histogram of 1000 random variables generated with $k = 2$ and $c = 5$.