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Original Research Article

The evaluation of some mechanical properties of materials by means of an identification method of a nonlinear dynamical system

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ABSTRACT

The determination of the dynamic properties of materials is a complicated process. Ordinary static tension or compression tests often reveal a dependence between the elastic properties and the deformation velocity (which in such tests has an initially fixed constant value). This means that the rheological model of materials may be complicated, often going beyond the generally used linear viscoelastic model.

The identification method presented in this paper can be helpful in determining the form of the rheological model of a material. The method consists in an appropriate analysis of the load-deformation dependences determined for a wide range of velocities of the motion of a concentrated mass attached to a flexible element made of the tested material.

The method is based on the rheological Zener model appropriately extended to enable the evaluation of mechanical material properties also at high strain rates. Therefore a solid fraction, a nonlinear spring function and a nonlinear damping function are additionally included in the model. The forms of the functions are not fixed a priori, but are developed by the presented method.

The method was verified on a numerical example as well as on a real material object.

1. Introduction

The viscoelastic properties of materials are generally tested under static or quasistatic loads at low deformation rates. Recently, numerous tests carried out on modern construction materials and also on biological materials (e.g. bones) show a strong dependence between Young's modulus and the deformation velocities, particularly when the latter are high [1–3]. Materials must be tested at high deformation velocities since they are often exposed to impact damage and in such cases (e.g. vehicle collisions) deformation velocities may be extremely high. Therefore materials need to be tested under fast changing dynamic loads. A typical example here is the piercing of a ballistic shield by a missile. Initially the shield deforms at a high rate, but subsequently the rate gradually decreases, ultimately reaching zero (when the missile stops in the shield). Hence the question arises: what Young's modulus value should be assumed for the analysis of this process? Consequently, it becomes necessary to find such a rheological

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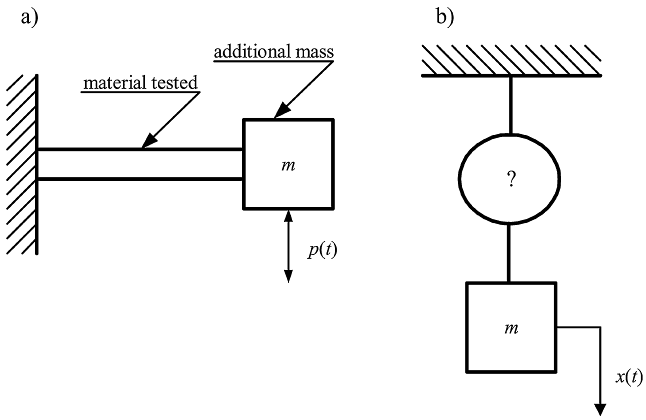

Fig. 1 – Illustration of adopted approach (b) for cantilever beam (a).

model of the material which will as accurately as possible describe the behavior of the latter in a wide range of deformation velocities. In the present study it was assumed that this could be done by constructing an identification method for simple single mass dynamic systems whose deforming component is made of the tested material. An example of such a system is a cantilever-wise fixed beam with concentrated mass m attached to its free end (Fig. 1).

If concentrated mass m is incomparably large relative to the mass of the beam, then the initial form of vibration is dominant, whereby one can assume the dynamic model of the whole system, as shown in Fig. 1b. The differential equation of the motion of mass m can be written in this general form:

$$m\ddot{x} + S(?) = p(t) \tag{1}$$

where $S(?)$ it the force with which the tested element (material) acts on mass m.

The problem of determining the mathematical form $S(?)$ as a function of relative velocity $\dot{x}$ and relative displacement x, for both free and forced vibration, has been described in numerous works. One should mention here the papers by Masri [4,5] and many other authors [6–10]. Usually a concrete form of function $S(x, \dot{x})$ is assumed and an appropriate identification procedure is derived for it, typically for a single-degree-of-freedom system (as, e.g., in [7] and the previous papers by the author [11–14]).

Some parametric identification methods are also developed for discrete multi-degree-of-freedom systems as well as for continuous systems. These methods are also often created for complex random excitations [15,16]. However some nonparametric identification procedures are being also developed (e.g. [4,9,12]). These procedures are important because their task depend on discovering the shape of the nonlinear elements which can be appeared in the real dynamical systems and, as it is known, the type of nonlinearity determines the complexity of the dynamic behavior of a complex system (e.g. [8]).

In contrast to the existing solutions, it is assumed here that exciting force $p(t)$ produces the displacement of mass m, proceeding at a constant velocity:

$$\dot{x}(t) = v_0 = \text{const.} \tag{2}$$

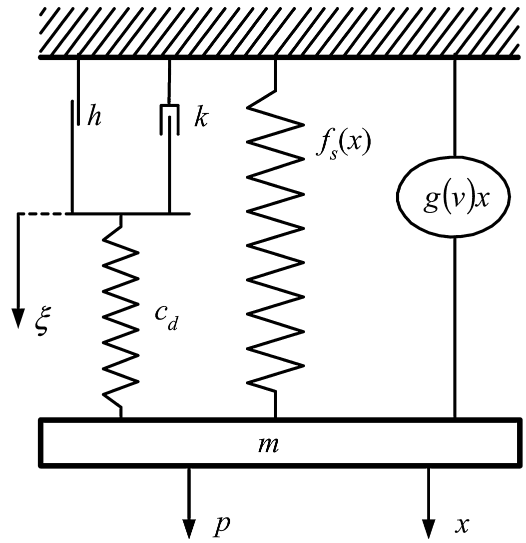

Fig. 2 – Scheme of adopted model.

i.e. similarly, as in the strength testing of materials. In order for the method to be universal enough for practical purposes, the rheological material model was assumed in the form of the extended Zener model (Fig. 2) in which the particular symbols stand for:k, c_d – constants describing the Maxwell element, h – a constant describing dry friction, $fs(x)$ – a displacement function in any nonlinear form, describing the reaction force of a purely elastic element, $g(v)$ – the so-called dissipative characteristic described by any nonlinear velocity function whose importance increases with the state of deformation of the material.

The aim of the proposed method is to determine constants h, k, c_d and functions $f_s(x)$, $g(v)$ from dependencies $p(x)$ experimentally determined for a series of constant velocities v_{01}, v_{02}, ... v_{0n}.

Since all the parameters of the assumed model are constant the method can be applied to materials whose properties are stable over time. This means that when the values of the properties clearly depend on, for example, temperature, the method cannot be used.

However, an important advantage of this method is that it is very versatile owing to the form of the assumed model (Fig. 2) incorporating the traditional Kelvin model (for $c_d = \infty$, $f_s(x) = cx$, $h = 0$, $g(v) = 0$) and the Maxwell model (for $h = 0$, $f_s(x) = 0$, $g(v = 0)$. As a result, the method can be applied to a very wide range of materials, especially in cases when Young's modulus depends on the strain rate.

Although Fig. 1 shows a cantilever beam subjected to bending, the method can be generalized to cover any type of material deformation. In the case of torsion, for example, variable x stands for an angle of torsion, m is a moment of inertia, $p(t)$ is a torque. Therefore, the method described below can be useful for any type of deformation.

2. Mathematical analysis of the model

The aim of the mathematical analysis is to derive the form of function $S(?)$ occurring in Eq. (1) and then to derive relations $p(x)$ under condition (2). The starting point are two differential