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Large deflections of nonlinearly elastic functionally graded composite beams

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ABSTRACT

The paper discusses governing differential equation for determining large deflections of slender, non-homogeneous beam subjected to a combined loading and composed of a finite number of laminae, which are made of nonlinearly elastic, modified Ludwick's type of material with different stress–strain relations in tension and compression domain. The material properties are varying arbitrarily through the beam's thickness. When the thickness of laminae is sufficiently small and the variation of mechanical properties is close to continuous, the beam can be considered as made of functionally graded material (FGM). The derived equations are solved numerically and tested on several examples. From a comparison of the results obtained and those found in the literature a good agreement was observed.

1. Introduction

The ability to compute deflections either for estimation of rigidity of an element and/or structure, comparison of theoretical and experimental results, computation of allowable deflections, or a post-buckling analysis, has always been desired. Large deflections of flexible elements have been in the center of attention to a number of researchers who tried to understand, model and determine their states. There exist many assumptions which gave rise to theories for modeling large deflections. Namely, for slender beams, where the influence of shear stresses and the inner axial force can be neglected in comparison to the dominating inner bending moment, Euler–Bernoulli beam theory is the most appropriate and frequently used. For thicker beams more accurate kinematic descriptions of the beams that consider the presence of shear stresses can be used, e.g. Timoshenko's or Reissner's description.

In recent years, effects of geometrical nonlinearities are being complemented by studies of material nonlinearities. In particular, Lewis and Monasa [1] and Lee [2] dealt with large deflections of thin cantilever beams of non-linear Ludwick type materials subjected to an end moment and combined loading consisting of uniformly distributed load and one vertical point load at the free end, respectively. Large deflections of a nonlinearly non-prismatic cantilever beam subjected to an end moment and static stability of nonlinearly elastic Euler's columns made from materials obeying the modified Ludwick constitutive law was investigated by Brojan et al. [3,4], respectively. Furthermore, in the works by Baykara

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et al. [5] and Brojan et al. [6], nonlinear bimodulus material was considered. Al-Sadder and Shatarat [7] developed a technique for a large deflection problem of a prismatic composite cantilever beam made of two different nonlinear elastic materials and subjected to an inclined tip concentrated force.

In the last two decades, demands for advanced materials that are capable of withstanding high temperature environment and exhibiting adequate mechanical performance, have stimulated the study, development and fabrication technologies of functionally graded materials (FGMs). With FGMs, where the material properties (e.g. Young's modulus, density, heat conductivity) of two or more constituents continuously vary as a function with respect to prescribed spatial directions, the desired performance of components can be tuned. Today FGMs have become widely used in aerospace, aircraft, automotive and civil structural, thermal, optical and electronic applications.

One of the first studies of modeling and design of multilayered and graded materials was published by Suresh [8]. Sankar [9] obtained an elasticity solution for a simply supported FGM beam. Furthermore, Zhong and Yu [10,11] presented analytical solutions for orthotropic functionally graded beams with arbitrary elastic moduli variations along the thickness direction under different boundary conditions. Using the displacement function method an analytical solution of a FGM beam with arbitrary graded material properties was investigated by Nie et al. [12]. A new beam element based on the first order shear deformation theory to study the thermoelastic behaviour of FGM beam structures was developed by Chakraborty et al. [13]. Li [14] gave a unified approach for analyzing static and dynamic behaviours of FGM Timoshenko and Euler–Bernoulli beams. Kang and Li [15,16] investigated the effects of depth-depended Young's modulus and the non-linearity parameters on the large deflections of the FGM beam. Similar problem was investigated by Kocatürk et al. [18], where Timoshenko beam theory and FEM are used. Soleimani and Saddatfar [19,20] presented large deflections of axially functionally graded beam using shooting method.

The present paper considers the problem of large deflections of slender, non-homogeneous cantilever beam subjected to a combined loading consisting of the distributed continuous loads and point loads at the free end. The material of which the beam is made is assumed to be nonlinearly elastic and only locally homogeneous. The mechanical properties are varying arbitrarily through the beam's thickness with different stress–strain relations in tensile and compressive domain.

The main focus of the paper is to derive the governing differential equations for determining large deflections of the beam composed of a finite number of laminae. Each lamina is in general characterized by their constant thickness and material properties. Some of the results obtained in this study are compared to those found in the available literature.

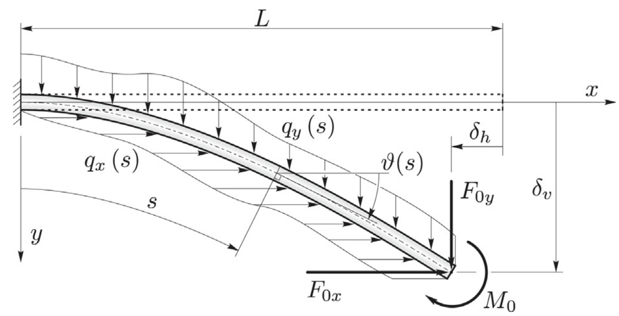

Fig. 1 – Deflected state of the cantilever FGM beam.

2. Definition of the problem

Consider a slender, initially straight elastic cantilever FGM beam of length L and uniform rectangular cross-section of thickness h and width b. The beam is subjected to the distributed continuous loads $q_x(s)$, $q_y(s)$ and point loads at the free end, i.e. F_{0x}, F_{0y} and M_0, see Fig. 1. This FGM beam is composed of n laminae which are different in general, each characterized by their constant thickness and material properties.

The mathematical model of the discussed problem is based on the elastica theory with the following assumptions:

- the material of which each lamina is made is assumed to be incompressible, isotropic, nonlinearly elastic and homogeneous. No slip condition between particular laminae is considered. They are rigidly bonded together;
- different nonlinear relations between the stress and strain in tensile and compressive domain are considered, see Fig. 3;
- the stress–strain relationship is assumed to be governed by the modified Ludwick constitutive model, mathematically described by the following expression

$$\sigma_i(\varepsilon) = \begin{cases} \sigma_{t,i}(\varepsilon) = E_{t,i}\left((\varepsilon+\varepsilon_{0t,i})^{1/k_{t,i}} - \varepsilon_{0t,i}^{1/k_{t,i}}\right) & \text{for } \varepsilon \geq 0, \\ \sigma_{c,i}(\varepsilon) = -E_{c,i}\left((-\varepsilon+\varepsilon_{0c,i})^{1/k_{c,i}} - \varepsilon_{0c,i}^{1/k_{c,i}}\right) & \text{for } \varepsilon < 0. \end{cases} \quad (1)$$

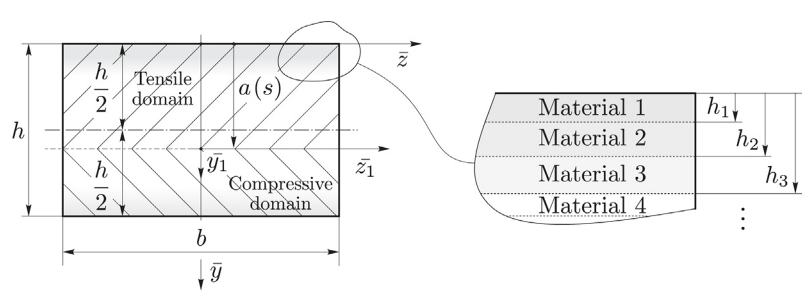

Fig. 2 – Cross-section of the FGM cantilever beam.