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An efficient nonlinear analysis of 2D frames using a Newton-like technique

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ABSTRACT

The Newton–Raphson method, which is based on the Taylor series and uses the tangent stiffness matrix, has been widely used to solve nonlinear problems. In this paper, a Newton-like algorithm is used for analyses involving geometric nonlinearity. This iterative technique that requires two initial guesses is known as two-point iterative method. In this method, a real function is assumed to approximate the tangent stiffness matrix of the structure. This paper, proposes an efficient function for reducing the computing time and, number of iterations in the Newton–Raphson method coupled with the two-point methodology. The computational nonlinear analysis on planar frames shows that the proposed strategy can reduce the computing time up to around 40%. Compared with the classic Newton–Raphson algorithm, the presented method proposes a methodology which also can reduce the number of iterations.

1. Introduction

Recently, nonlinear analysis has started to replace linear analysis for many structural engineering applications, including in the analysis of 2-D frames. In parallel with the development of more intricate analysis methods, the speed and memory capacity of computers continue to increase each year. Making it possible to do analysis and design techniques that in the recent past were impossible.

Linear and nonlinear analysis can be conducted with consideration of small or large deflections. Analysis with consideration for large deflections expresses that the structural configuration deforms obviously, which results in a change of originally assumed displacement and forces directions. In geometric nonlinear or second-order elastic analysis, equilibrium equations between internal forces and external loads are formulated on the deformed configuration of the structure. In this paper, the large deflection formulation is used during elastic conditions.

For three decades now, nonlinear elastic and inelastic analysis of frame structures has been a topic of considerable research. Kassimali [1] presented a numerical procedure for the large deformation analysis of elastic–plastic plane frames. Saffari et al. [2] introduced a algorithm for nonlinear analysis which can decrease number of iterations and computing time. Tabatabaei and Saffari [3] studied large strain analysis of planar frames using a normal flow algorithm. Scott and Filippou [4] considered response gradient for nonlinear elements under large displacements. Kassimali and Garcilazo [5] proposed a procedure for large displacement analysis of elastic plane frames subjected to temperature changes. Tabatabaei et al. [6] utilized the Newton–Raphson method along the flow path normal in pushover analysis of frames.

Of all numerical computation, systems of nonlinear equations are maybe the most laborious to solve. There are various techniques for solution of these equations. For a nonlinear problem, the solution of the nonlinear system of equations usually takes up most of the computational time.

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Many iterative methods have been proposed for improving Newton–Raphson method [7,8]. However, many methods depend on the higher derivatives from the computing procedure that limits their practical application because of intrinsically laborious and vast essence of the computation involved.

In recent years, there have been many developments in making up iterative algorithms designed to improve usage of the Newton–Raphson method while at the same time, not needing the computation of second derivatives (Hessian matrix) for solving the nonlinear equations (9,10). Iterative algorithms can be categorized as either depending on the one-step or the two-step category. Two-step methods have been proposed by incorporating the Newton–Raphson algorithm with another one-step approach [7]. This method has adaptation to predictor-corrector methods. Multipoint iterative solvers belong among such strong methods for solving nonlinear equations which can defeat the theoretical restrictions of one-point algorithms regarding the convergence order and computational efficiency [9]. Saffari and Mansouri [10] applied two-point method for geometrically nonlinear analysis of structures, especially in the case of trusses. However, the method traces the equilibrium path until limit point. A Newton-like scheme is developed for frames and a new function is proposed to increase the convergence rate of analysis. Furthermore, limit points can be passed using the presented algorithm.

2. Geometrical nonlinear analysis of planar frame

2.1. Primary information

Fig. 1 illustrates a beam-column element of cross-sectional area A, length L, Young's modulus E and second moment of area I, subjected to member end forces $\{F_1, F_2, F_3, F_4, F_5, F_6\}^T = \{F\}$ in global coordinates. For the 2D frame element in its initial form, the global nodal coordinates are (X_1, Y_1) for node 1 and (X_2, Y_2) for node 2. For the plane frame element in its current formation the global nodal coordinates are (X_1+V_1, Y_1+V_2) for node 1 and (X_2+V_4, Y_2+V_5) for node 2.

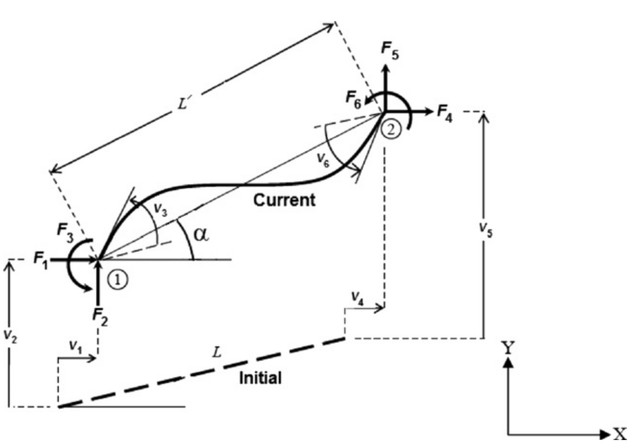

Fig. 1 – Member forces and deformations in global coordinates.

The current length of the plane frame member is:

$$L' = \sqrt{\{(X_2 - X_1) + (V_4 - V_1)\}^2 + \{(Y_2 - Y_1) + (V_5 - V_2)\}^2} \quad (1)$$

In Fig. 2 a plane frame member is shown subject to member forces in local coordinates. These forces entail an extension δ, and end rotations θ_1 and θ_2 at the ends 1 and 2, respectively. The sign conventions for forces and displacements are also illustrated in the Fig. 2.

The expressions for element internal forces (Q=Axial force; M_1, M_2=end moments) and system tangent stiffness matrix can be found in [11]. Obviously, shear forces depend on the end moments of the member.

The transformation relation between an element's local forces, $\{S\} = \{M_1\ M_2\ Q\}^T$, and its global end forces $\{F\}$ (see Fig. 1) can be expressed as:

$$\{F\} = [B]\{S\} \quad (2)$$

in which the transformation matrix is:

$$[B] = \frac{1}{L'}\begin{bmatrix} -n & -n & mL' \\ m & m & nL' \\ L' & 0 & 0 \\ n & n & -mL' \\ -m & -m & -nL' \\ 0 & L' & 0 \end{bmatrix} \quad (3)$$

with

$$m = \cos\alpha \; n = \sin\alpha \quad (4)$$

L' and α refer to the length and orientation, respectively, of the chord of the element in its deformed configuration, as shown in Fig. 1.

2.2. System equilibrium equations

The equations of equilibrium of the system can be expressed as follows:

$$\{f(\delta)\} = \{P\} \quad (5)$$

which $\{f\}$ is the resultant of the nodal internal forces and $\{P\}$ presents external nodal forces. The element force deformation relationships interpret that $\{f\}$ is a highly nonlinear function of $\{\delta\}$. The differential formation of Eq. (5) is:

$$[\tau]\{\Delta\delta\} = \{\Delta P\} \quad (6)$$

here $\{\Delta\delta\}$ and $\{\Delta P\}$ are increments of displacement and load, respectively.

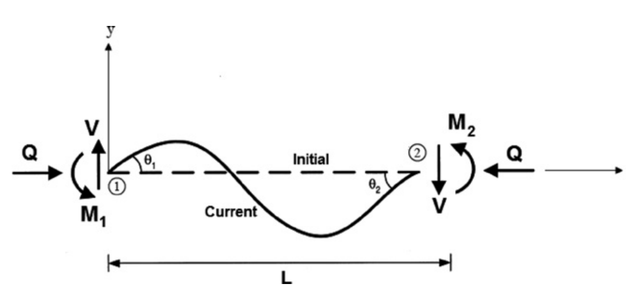

Fig. 2 – Member forces and deformations in local coordinates.