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Numerical aspects of application of FORM in node snapping truss structures

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ABSTRACT

In the present paper numerical aspects of application of first order reliability method FORM in node snapping truss structures are considered. The number of the programme FEM runs and the number of iterations indicate that the proposed method provides accurate and computationally efficient estimation of the probability of failure. The study shows comparative analysis of the reliability index computations by the method of FORM, SORM, importance sampling, Monte Carlo and estimate computation error. The interesting problem is evaluation of the reliability index sensitivity with respect to random variables, mean value, and standard deviation.

1. Introduction

In the traditional structural design, the deterministic values of the design parameters are used. The structural safety related to variation in structural parameters is ensured by choosing conservative values of those parameters, and also by accounting for safety factors in the equations of the limit states. The use of random variables to represent structural parameters makes it possible to explicitly account for randomness in the design process. Consequently, it is possible to construct a mathematical model to estimate the probability of a specific behaviour of a structure. Due to the fact that available test results are not sufficient to perform probabilistic analyses (disregarding, at this point, the complexity of such analyses), engineers tend to avoid using probabilistic methods. It also refers to probabilistic numerical methods, the complexity of which lies, in fact, in that of computer programs. An additional effort on the part of the program user is required when the data are characterized with two parameters (expected value and standard deviation) instead of one that is necessary in the deterministic methods.

It should be emphasized that the structural reliability issues are actually related to real-life situations. Eurocodes, introduced in 2010 and binding throughout Europe, are, to a far extent, based on the so-called probabilistic approach. In accordance with Eurocodes, design standards and loading standards should be formulated in such a way so that structure members dimensioned on such basis would ensure appropriately high level of reliability. The most objective measure of a failure occurrence is its probability P_f. In the paper the Hasofer–Lind reliability index β is assumed as the

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reliability measure; $P_f = \Phi(-\beta)$ (Φ is the standard normal cumulative distribution function). Establishing the final reliability level for standards that are newly developed is still at the stage of investigations. It should be remembered that different indexes are relied on when a slab is designed, or a second beam, or a main beam supporting the second beam. Every new computational method involves laborious tests that have to be repeated many times to validate the adequacy of the method for a given engineering problem. The present study provides only a minute contribution to investigations, an enormous amount of which need to be performed to correctly estimate the final reliability index for different types of structures.

The context for this paper is the growing need for reliability analysis in conjunction with advanced numerical computations by means of the finite element method FEM. This type of analysis is useful in many civil engineering applications to account for uncertainties by estimating the probability of response events, with all input parameters given as random variables. Compared with traditional structural reliability applications, such analyses carry unique opportunities and challenges. In this paper emphasis is placed on the first-order reliability method FORM due to its appealing tradeoffs between accuracy and computational efficiency. Computational cost is of paramount concern when FEM is involved. The time required to run one FEM analysis may be in the order of hours. This places strict constraints on the computational cost of the reliability method. FORM typically requires 5–10 evaluations of the limit state function and its derivative with respect to the random variables to find the design point thus obtaining the probability estimate. It is noted that a number of alternative reliability methods are available, including several sampling procedures. However, under favourable conditions, FORM provides a supreme balance of computational cost and accuracy. Nevertheless, it is imperative that the analyst recognizes the problem definitions that are either infeasible or have the potential for substantial errors. In fact, the convergence and the accuracy of FORM are critically dependent on the characteristics of the limit state function, which in turn depends on the characteristics of the FEM responses that enter in the function.

The utilization of FEM in reliability applications has increased in recent years. A partial list of contributions to the FEM based reliability methods includes those by Der Kiureghian and Taylor [1], Liu and Der Kiureghian [2], Gutierrez et al. [3], Der Kiureghian and Zhang [4], Imai and Frangopol [5], Der Kiureghian and Haukaas [6] and others. The term finite element reliability analysis is employed to characterize the methodology. The objective in this type of analysis is essentially to compute the probability of rare response events, which are defined in terms of limit state functions.

The scope of this study is limited to research numerical aspects of application FORM in node snapping truss structures. Consequently this study does not include the effect of non-normal probabilistic distribution functions, and probabilistic boundary conditions. These problems are analysed by the author in the paper [7]. The feasibility and accuracy of FORM has traditionally been studied in the context of "explicit" limit state functions, i.e., with algebraic limit state function expressions [8–11]. The present study considers the problems of stability and reliability of truss structure susceptible to stability loss from the condition of node snapping. In this situation we must analyze not explicit limit state function. Non-linear geometrical relations are defined in Lagrange description. Stability analysis of structure is done by means of the finite element method. In the paper the constant arc length method [12,13] and the current stiffness parameter method [14,15] are used for the determination of equilibrium path. Let us now ask a question, what is the advantage of the inclusion of reliability analysis methods to the analysis of stability? The answer is the following, using methods of reliability analysis, moving along the equilibrium path of structure, we can determine the level of failure probability when we approach the critical point. We can estimate reserve of safety.

2. Finite element method FEM implementation of FORM

In early applications of reliability analysis methods, it was accepted that the limit state function is an explicit function of random variables. Such functional dependency can be realized only for very simple examples. In practical realizations, this dependence is not explicit and it is determined using numerical procedure, e.g. the finite element methods FEM. This article presents the communication between the reliability analysis programme STAND and external FEM programme KRATA. The programme STAND was developed in the Institute of Fundamental Technological Research of Polish Academy of Science by J. Knabel, K. Kolanek, V. Nguyen Hoang, R. Stocki, R. Lasota, and P. Tauzowski [16,17]. The programme KRATA was developed by the author.

Limit state functions are expressed in terms of load effects such as stresses, deformations, and displacements, whereas statistical information is given for uncertain basic variables, such as material properties, loads, and geometry parameters. Let the vector $\mathbf{s}$ (external variable) and $\boldsymbol{\nu}$ (basic random variable) respectively denote these two sets of variables. In the presented examples basic random variables are load, axial stiffness of bars and node coordinates; external variables are displacements and load multiplier. For most structures, the relation $\mathbf{s} = \mathbf{s}(\boldsymbol{\nu})$ is not explicit and can be evaluated through FEM. The object in reliability analysis is estimating the failure probability P_f.

$$P_f = \int_{\Omega_f} f_{\boldsymbol{v}}(\boldsymbol{v}) d\boldsymbol{v}, \quad \Omega_f = \{\boldsymbol{v} : g(\mathbf{s}(\boldsymbol{v}), \boldsymbol{v}) \le 0\} \tag{1}$$

where $f(\boldsymbol{\nu})$ is the joint probability density of $\boldsymbol{\nu}$, $g(\mathbf{s}(\boldsymbol{\nu}),\boldsymbol{\nu})$ is the limit state function defined such that $g(\mathbf{s}(\boldsymbol{\nu}),\boldsymbol{\nu}) \le 0$ denotes the failure domain Ω_f.

FORM is aimed at approximately evaluating the given integral. First, the basic variables are transformed to standard normal space through a transformation $\mathbf{y} = \mathbf{y}(\boldsymbol{\nu})$. Der Kiureghian and Liu [18] suggested a probability transformation which is particularly useful in the finite element reliability methods. In this method, a joint distribution model, originally introduced by Nataf [19], with prescribed marginal distributions and correlation matrix was proposed. In the FORM method, one searches the point on the limit state surface nearest to the origin in the standard normal space by solving