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Numerical homogenization by using the fast multipole boundary element method

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The paper concerns the numerical homogenization of non-homogeneous linear-elastic materials by using the fast multipole boundary element method (FMBEM). Application of the FMBEM allows for the analysis of structures with larger number of degrees of freedom (DOF) in comparison to the conventional collocation BEM, which has at least the quadratic complexity. The FMBEM is convenient in the numerical homogenization by modelling of complex representative volume elements. In this article two examples of homogenization are presented. The first one concerns a porous material, and the second one – a composite material. The obtained results are in good agreement with analytical, semi-empirical and empirical models, and also with numerical results presented by other authors.

Keywords: *numerical homogenization, fast multipole boundary element method, effective elastic constants*

1. Introduction

The design of elements made of non-homogeneous materials (e.g. porous or composite ones) requires the knowledge of effective (homogenized) elastic constants. For the determination of the properties one can apply analytical, empirical or numerical methods. In the analytical methods some assumptions are made and thus the area of application is limited to relatively simple geometry of microstructure and a small volume fraction of pores, inclusions, etc. The empirical methods require the preparation of specimens which are subsequently tested for the searched properties. This results in relatively high costs. A meaningful alternative for the above described methods is the numerical analysis by using the representative volume element (RVE) concept [1].

There are several methods (or groups of methods) used for the numerical analysis of physical structures and phenomena, i.e.: the finite difference method (FDM), the finite element method (FEM), the boundary element method (BEM) and the group of meshless methods. From the point of view of geometry modelling (irregular boundaries, multiply connected domains and subdomains), which is important in the context of the numerical homogenization, the FEM and the BEM can be considered more effective than other methods. Furthermore, the BEM can be more efficient than the FEM because in many cases it requires only the discretization of boundaries of the analyzed domain. However, the complexity of the conventional collocation BEM is $O(N^2)$, where N is the number of degrees of freedom (DOF) of the analyzed structure. This affects the possible maximum size of analyzed structures, which usually does not exceed several thousands of DOF. The analysis of large structures by the BEM can be extremely time-consuming. In order to reduce the complexity the fast multipole method (FMM) can be used [2]. The fast multipole BEM (FMBEM) has the linear complexity and can be applied to the analysis of much complex structures in relation to the conventional BEM.

Several authors performed the static analysis of linear-elastic structures with holes, inclusions or cracks, which modelled RVEs or unit cells of non-homogeneous materials, by using different versions of the BEM. Firstly a short review of selected papers concerning the conventional BEM applications will be given. Eischen and Torquato [3] analyzed 2-D unit cells containing inclusions forming the hexagonal array. The effective material properties for a number of inclusion-matrix elastic constants' ratios and volume fractions of inclusions were given. Hu et al. [4] analyzed perforated plates with randomly or normally distributed identical holes by the conventional BEM. Effective moduli of the structures were determined and compared to values obtained by several analytical models. Due to a relatively large number of degrees of freedom (about 8 000) a special technique, consisting in the processing of the main matrix of the system of equation partially, was applied to overcome the limitation of the computer memory. Yao et al. [5] proposed a BEM formulation for the linear-elastic analysis of plates containing many identical inclusions. Yang and Qin [6] used the concept of unit cell for the determination of effective elastic constants of composites, modelled as 2-D structures containing rigid circular inclusions. The results were compared to finite element method results.

The structure size limitation encountered in the analysis by the conventional collocation BEM pulled researchers to use improved versions of the BEM, with reduced complexity. One of the first papers concerning the analysis of non-homogeneous structures by a multipole BEM was by Yamada and Hayami [7]. They applied a multipole BEM code based on the algorithm of complexity O ($N \log N$), using the multipole expansion only (without the local one), to the analysis of a rectangular plate with many holes. They analyzed structures with up to 900 constant elements. For such numbers of DOF, the multipole algorithm appeared to be less effective than the conventional one. Greengard and Helsing [8] applied the Sherman formulation for the analysis of linear-elastic unit cells containing inclusions of different shapes. They assumed periodic boundary conditions to determine effective elastic constants of the material. Helsing [9] applied the FMBEM and the complex potential formulation to the determination of effective elastic properties of 2-D RVEs containing many cracks. Helsing and Jonsson [10] proposed a new complex potential BEM formulation. They applied the new method in combination with the FMM to the evaluation of effective properties of 2-D structures containing many circular and elliptic holes. Yao et al. [11] applied their formulation for many identical inclusions with the FMBEM, introduced in [5], to the evaluation of elastic constants of composites with the direct bonding between inclusions and matrix, and also with interphases between inclusions and

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