ARCHIVES OF CIVIL AND MECHANICAL ENGINEERING

Vol. XI 2011 No. 3

Improved implementation of the extended finite element method for stress analysis around cracks

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Although the extended finite element method (XFEM) allows for modelling arbitrary discontinuities, its low order elements often means that frequent improvements on accuracy are required. The generalized finite element method (GFEM), the extension of the conventional FEM, improves the approximation accuracy of the FEM by introducing generalized degrees of freedom and re-interpolating nodal degrees of freedom. This paper enhances the implementation of the XFEM for stress analysis around cracks by coupling the GFEM and XFEM. The generalized node shape functions are used in a cluster of nodes around the cracks, and the conventional finite element shape functions are adopted at nodes outside the cracks, thereby reducing costs and improving the accuracy of stresses in the vicinity of the cracks. Several numerical examples show that the proposed approach generates higher accuracy for stress intensity factor computations at affordable costs.

Keywords: *XFEM, GFEM, crack, stress intensity factor*

1. Introduction

Finite element methods (FEM) have been widely used in research and engineering. However, because of the fact that standard FEMs are based on piecewise polynomial approximations, they are not well suited to solving discontinuity problems. To overcome this deficiency, the extended finite element method (XFEM) was introduced in 1999 [3].

The XFEM is a numerical method used to simulate the discontinuity within the standard finite element framework [29]. In this method, the standard displacement-based approximation is enriched by incorporating discontinuous fields through the partition of unity method [24]. This enables the geometry of discontinuity to be independent of the finite element computation mesh; thus, it is very convenient for discontinuous problem analysis, especially for moving discontinuous problems. In the past decade, numerous efforts have been made to improve or apply the original XFEM for simulations of fractures and other discontinuous phenomena. Studies where the XFEM is used in fracture problems are [3, 29] and [14, 30, 38] for the two-dimensional and three-dimensional elastic crack growths, respectively; [17–18] for cohesive crack propagation; [22, 37] for bi-material interface crack; [25, 27–28] for dynamic crack propagation; [42] for fatigue crack propagation; and [10] for elastic-plastic crack growth. The surveys of the XFEM for computational fracture mechanics were provided by several scholars in [20, 32, 44]. The XFEM is also successfully used in other areas of computational mechanics such as fluid–structure interactions [13], phase transformations [15], shear band propagation [1], evolution of dislocations [31], and biofilm growth [8]. The XFEM/GFEM was reviewed with an emphasis on applications in materials science problems by Belytschko et al. [4] and highlighting methodological issues by Fries et al. [12].

One difficulty with the XFEM is that it sometimes fails to exhibit the desired accuracy. Numerous studies have been directed toward improving the accuracy of the fields near the crack. Iarve [16] replaced the Heaviside step function with a higher-order polynomial B-spline shape function approximation. Stazi et al. [35] used higher-order background elements for linear elastic fracture mechanics that is able to obtain higher accuracy for curved cracks. Liu et al. [22] enriched the crack tip node with the leading, as well as higher-order terms of the asymptotic crack tip fields, in an attempt to significantly improve the accuracy of the directly determined stress intensity factors (SIFs). Recently, elastic fracture work was extended to thermoelastic fractures [45], in which the benefit of including higher-order terms is greater for thermoelastic problems than for either purely elastic or steady state heat transfer problems. Béchet et al. [2] expanded the range of the crack tip enrichment area by a "geometrical" instead of a "topological" enrichment. Numerical accuracy was consequently improved.

The standard formulation of the XFEM approximations leads to problems in blending elements [5], and the existence of blending elements reduces accuracy. Fries proposed "the corrected XFEM" employing a new definition of enrichment [11]. The corrected XFEM showed the effective improvement of accuracy reduction caused by the blending elements. Later, the theoretical validation of "the corrected XFEM" was determined based on the concept of the PUFEM approximation [34].

For dynamic crack propagation in the context of the XFEM, recent work on time-dependent tip enrichment [28] and release of the crack tip element [26] showed some improvement in XFEM accuracy in SIF computation. The enrichment of the XFEM approximations by mesh-free approximations was reported in Reference [25]; this approach leads to higher accuracy in SIF computation, and to the capability to capture the branching point of a propagating crack from the stresses.

An alternative method of enhancing the quality of the stress field near the crack tip is adaptively driven by error estimation developed in [9, 33, 41].

The generalized finite element method (GFEM) was adopted by the Texas school [23] and the Institute of Mathematics of the Chinese Academy of Sciences [21] in 1995–1996. Based on the partition of unity method, the GFEM improves the approximation accuracy of the FEM or achieves the special approximation in particular problems by introducing generalized degrees of freedom (DOF) and re-interpolating nodal DOF. The first GFEM studies involved global enrichments of the approximation space; later, local enrichments for singularities at sharp corners were also developed [7]. Strouboulis et al. [36] systematically studied the GFEM by the combination of