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Original Research Article

Spring Method for modelling of particulate solid composed of spherical particles and weak matrix

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ABSTRACT

In the present paper, a possibility of an approximation of elastic particulate composite with a network of elastic springs that undertake only axial forces is considered. It is assumed that the springs are equivalent to two hemispheres interacting through a weaker interface member. In a frame of the suggested approach, the description of the composite is limited to translational degrees of freedom, therefore, only a normal interaction between the spheres was considered. The methodology for calculation of the axial stiffness of the elastic springs and obtained solutions of the stiffness in explicit form are the main novelty of the article. A comparison of the stiffnesses of the springs obtained by the proposed methodology and by the three dimensional Finite Element Method (FEM) has shown a good agreement between them in a wide range of the ratio of the modulus of elasticity of the particles and matrix at four different distances between surfaces of the particles. A possibility of the approximation of particulate composite by springs was tested and discussed in details by comparing results of a mechanical response of a sample (under three different loading cases) modelled as a three dimensional solid and as a system comprised of the springs. The solutions were obtained by the FEM.

Notations

{...} is a set

u_x and u_y are displacements of a point along to axes x and y, respectively

$||\mathbf{u}|| = \sqrt{(u_x^2 + u_x^2)}$ is total displacement of a point

$D_{r,I}$, $D_{r,II}$ and $D_{r,III}$ are elastic constants of corresponding part of CE, $r \in \{i, j, b\}$ in uniaxial, biaxial and three axial stress states, respectively

D_r is an elastic constant of particle or bond element

E_i, E_j and E_p are elastic moduli of particles i, j and p, respectively

k_i, k_j and k_b are stiffnesses of a half of the particle j, j or the interface member, respectively

K_s and $K_{s,lim,0}$ are stiffness of the CE according to proposed methodology and its upper bound

$K_{s,lin}$ is linear stiffness of a CE

$K_{s,pot}$, $K_{n,H}$ and K_m are stiffnesses of CE according to Potyondi, Hertz and Pilkavičius et al.

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L_c is a distance between surfaces of interacting particles,
min{·} and max{·} are minimum and maximum element of a set {·},
R_i, R_j *and* R_b are radii of particles *i* and *j*, and the interface member, respectively
u is displacement
υ_i, υ_j and υ_p are Poisson's ratios of particles *i*, *j* and *p*, respectively
x, *y* and *z* are coordinate axis
β is a factor for the stiffness of a particle
Δl is displacement of centres of the particles or shortening of the CE
SM stands for the developed Spring Method
FEM stands for Finite Element Method
1D, 2D and 3D stand for one, two and three dimensions
3DS1, 3DS2 and 3DS3 stand for designations of samples of 3D FEM models
CE stands for connecting element
SMS1, SMS2 and SMS3 stand for the samples of the Spring Method
LC1, LC2 and LC3 stand for three loading cases: tension, shear and two-dimensional tension with shear

1. Introduction

In the past decade, many researchers have investigated numerically macroscopic behaviour of particulate composites consisting of particles embedded in a weaker matrix. Due to high heterogeneity of such composites, the direct application of the FEM is impeded. There are other numerical techniques that may be helpful for modelling of these composites (i) lattice models based on approximation of the composite by a network of linear springs (rods) and (ii) the Discrete Element Method (DEM). Both approaches require knowledge of effective mechanical properties of constituent parts and constituent laws of the normal interaction of particles embedded in a weaker matrix.

The earliest lattice models of 2D continuum were described by equality in terms of normal and shear spring stiffness. The explicit expressions for these models were probably given first by Sawamoto et al. [1], and then theoretically derived by Griffiths et al. [2], Hrennikoff [3], Kawai [4] and Herrmann et al. [5]. Simulations, employing network of classical Bernoulli-Euler beams for 2D problems are presented in the earlier work of Herrmann et al. [5], and later by Bolander et al. [6], for 3D problems by Lilliu and van Mier [7], and by Liu et al. [8,9]. The Timoshenko type beams are used by Karihaloo et al. [10] and by Ibrahimbegovic and Delaplace [11]. Enhanced 3D lattice beam models are considered by Kozicki and Tejchman [12], Andre et al. [13], Patyondy et al. [14], and Rojek et al. [15].

Inhomogeneity of a solid material can be considered employing disordered lattice or structural network as in the approach developed by Patyondy and Cundall [16]. They have assumed that the stiffness of the interface material can be incorporated into the stiffness of springs representing contact interaction between particles. An analysis of the heterogeneous structure of the material over different length scales is highly complicated. The simplest approach to model heterogeneous material on a micro scale is to assign different properties of materials to each finite element (Lilliu and van Mier [7]).

The paper is aimed to enhance an approximation of particulate composite by the structural network of spring elements. The attention is mostly focused on the deformation behaviour of two spherical particles that contact via a weaker bond member. The attention is addressed mainly to evaluation of the overall bond properties in terms of the elasticity modulus and the initial separation distance. A model of one-dimensional connecting element (CE) is developed to capture the normal interaction of the particles and to transmit data of meso-scale heterogeneity to a macro model. In the present article, only the normal interaction was taken into account. Validity of the developed model of CE was examined against a continuum approach employing the FEM.

2. Modelling concept

Particulate composite is approximated by a structural network that is comprised of one-dimensional connecting elements (CEs) with the nodes in the centres of the spherical particles (Fig. 1a). Two vicinal spheres interact with each other directly via an interface member that is comprised of a representative volume of the matrix (Fig. 1b). The CEs that undertake only axial forces F_n of the normal interaction are assumed to be one dimensional springs connecting centres of the two vicinal spheres (Fig. 1c). The CEs are characterized only by their length and an axial stiffness, thus the area and geometry of the cross-sections of the CEs are not specified. All the quantities related to particles are denoted by subscripts *i*, *j* or $p \in \{i, j\}$ and quantities related to interface member are denoted by a subscript *b*. An analogy with three sequentially connected

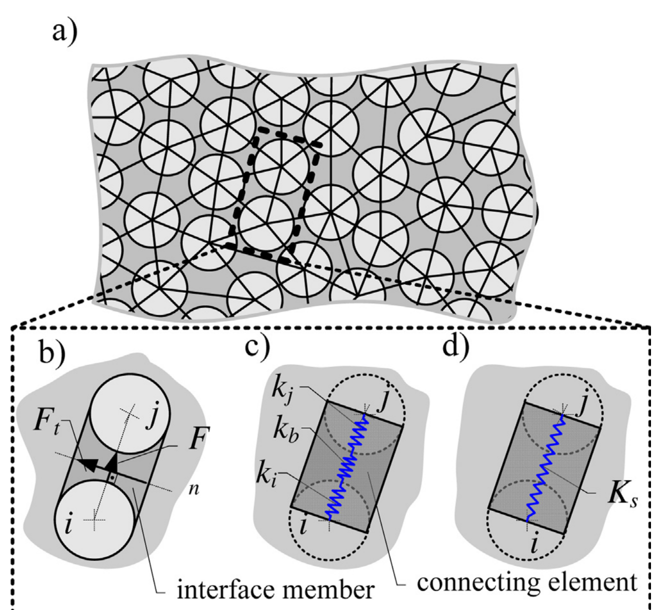

Fig. 1 – Discretisation concept: discrete model with non-regular lattice (a); binary normal interaction of two spheres through a conditional interface member (b); (c) and (d) represent the connecting element and the springs that correspond to the spheres, interface member and the entire connecting element, respectively.