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Original Research Article

Nonlinear motion characteristics of microarches under axial loads based on modified couple stress theory

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ABSTRACT

The aim of the present study is to investigate the geometrically nonlinear size-dependent bending as well as resonant behaviour over the bent state of a microarch under an axial load. In particular, an axial load is applied on the system causing the initial curvature to increase by giving rise to a new bent configuration. A distributed harmonic transverse force is then exerted on the microarch and the nonlinear resonant response of the system over the new deflected configuration is investigated. The nonlinear partial differential equation of motion is obtained via Hamilton's principle based on the modified couple stress theory. The equation is discretized into a set of nonlinear ordinary differential equations through use of the Galerkin scheme. The pseudo-arclength continuation technique is then applied to the resultant set of ordinary differential equations. First, for the unforced system in the transverse direction, the axial load is increased and the new deflected configuration of the system is plotted versus the axial compression load; the nonlinear resonant response over the deflected configuration is then investigated through constructing frequency–response and force–response curves.

1. Introduction

Microbeams are found in many micro devices such as in microswitches, biosensors, microelectromechanical devices, and pressure sensors [1–7]. As reported in experimental investigations [8–13], the deformations of the microscale elements are highly size-dependent which could not be predicted by classical continuum theories. As a result, new continuum theories, such as the strain gradient and modified couple stress theories, are developed to predict the size-dependent behaviour theoretically.

The literature concerning the statics and dynamical behaviour of *initially straight microbeams* can be categorized

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into two general classes of studies. The first class [14–17] examined the response of the system in the absence of the axial load, i.e. about the *trivial* equilibrium state. The second class [18] investigated the motion characteristics of the system when the axial load is exerted on the system causing a divergence to a new non-trivial configuration; the dynamical behaviour is then examined over the new non-trivial equilibrium state. In many applications, the microbeam is axially loaded, thus the second class of investigations can better predict the motion characteristics of the system.

The literature related to the first class of analysis is reviewed briefly in this paragraph; for example, Ma et al. [19] and Asghari et al. [20] employed the modified couple stress theory in order to investigate the effect of the length-scale parameter on the free oscillations of a Timoshenko microbeam. Roque et al. [21] examined the static deflection of a composite Timoshenko microbeam on the basis of the modified couple stress theory. The effect of a thermal loading on the oscillations of a bilayer microbeam was analyzed by Ross et al. [22] by means of both experimental and theoretical tools. Ke and Wang [23] examined the size effect on the stability of a functionally graded microbeam on the basis of the modified couple stress theory. Wang et al. [24] contributed to the field by examining the size-dependent oscillation behaviour of a cylindrical microbeam. Asghari et al. [25] analyzed the nonlinear oscillations of a Timoshenko microbeam based on the modified couple stress theory.

The second group of investigations is not as large as the first one; first, the papers in which a linear mathematical model is employed are reviewed. Ansari et al. [26] investigated the size-dependent buckling of a functionally graded microbeam based on the strain gradient theory. Şimşek and Reddy [27] contributed to the filed by developing the equations of motion of a higher-order microbeam on the basis of the modified couple stress theory for the buckling analysis of a functionally graded microbeam. Ke et al. [28] analyzed the thermo-mechanical oscillations and buckling of a microbeam based on the modified couple stress theory. The onset of buckling for microbeam under axial load was predicted by Akgöz and Civalek [29], employing the strain gradient and modified couple stress theories. The nonlinear investigations in this group of analysis is very limited; the postbuckling configuration of a microbeam and the nonlinear natural frequencies were obtained by Xia et al. [30] employing a single-mode Galerkin approximation. Moreover, Mohammadi and Mahzoon [31] examined the thermal effects on the nonlinear postbuckling configuration of microbeams based on the modified strain gradient theory.

As stated earlier, both the first and second groups of papers reviewed above are concerned with *perfectly straight* microbeams. However, in some applications, such as in microshutters, microvalves, bandpass filters, and microswitches, the microbeams are *initially curved*, i.e. forming *microarches*. Moreover, due to manufacturing imperfections, it is very likely that the produced microbeam has an initial curvature even if it is intended to be perfectly straight. The literature regarding initially curved microbeams (i.e. microarches) is not well developed; the current paper is the first to examine the nonlinear behaviour of a microarch, under axial and transverse forces, taking into account the length-scale parameter. In other words, to the authors' best knowledge, there is no investigation in the literature which examined the *nonlinear forced response of a microarch under an axial load based on the modified couple stress theory*. To this end, the size-dependent nonlinear equation of motion is obtained, via an energy-based technique [32–34], and then discretized into a high-dimensional system with a large number of degrees of freedom. In order to be able to solve this large number of equations numerically, efficient computer codes with optimized run-time and accuracy are programmed to obtain the static deflection and the dynamic response over the deflected configuration.

2. Model development via Hamilton's principle

Fig. 1 shows a simply supported microarch of length L, flexural stiffness EI, and axial stiffness EA under an axial load p in the x direction; as the axial load is increased, the microarch deflects to a new deflected configuration. The system is also subject to a distributed harmonic excitation load per unit length, $F(x)\cos(\omega t)$, in the z direction. x and z denote the axial and transverse coordinates, respectively, while $w(x, t)$ and $w_0(x)$ represent the transverse displacement and the initial curvature, respectively. The equation governing the motion of the system is derived based on the following assumptions: (1) the potential energy of the system is obtained based on the modified couple stress theory; (2) shear deformation and rotary inertia are neglected; (3) the cross-sectional area of the microarch is assumed to be uniform along the entire length; (4) there is a slight initial curvature with the amplitude of $w_0(x)$; (5) the nonlinearity considered is geometric due to the mid-plane stretching [35]; (6) it is assumed that axial load p is initially applied to the system causing a bending in the microarch by giving rise to a new deflected configuration and then the external transverse excitation force is applied to system.

Based on the modified couple stress theory [36], the strain energy of a system occupying region V can be written as

$$U = \frac{1}{2}\int_V \{\sigma : \varepsilon + \mathbf{m} : \boldsymbol{\chi}\}\, dv, \tag{1}$$

in which $\boldsymbol{\sigma}$, ε, $\mathbf{m}$, and $\boldsymbol{\chi}$ represent the stress, strain, deviatoric part of the couple stress, and the symmetric curvature tensors, respectively.

The symmetric curvature tensor $\boldsymbol{\chi}$ is given by

$$\boldsymbol{\chi} = \frac{1}{2}\left(\nabla\boldsymbol{\theta} + (\nabla\boldsymbol{\theta})^T\right), \tag{2}$$

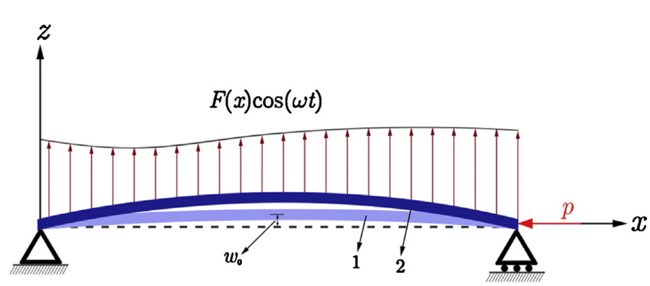

Fig. 1 – Schematic representation of a microarch subject to a transverse distributed harmonic excitation force over the bent configuration (due to an axial load). Number 1 is the microarch before bending; number 2 is that after bending.