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2D tolerance and asymptotic models in elastodynamics of a thin-walled structure with dense system of ribs

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ABSTRACT

The object of analysis is a plane structure reinforced by a system of thin parallel-distributed ribs. It will be assumed that the number of the ribs is very large. The thickness of neighbouring ribs can smoothly change. The aim of contribution is to derive 2D-macroscopic mathematical models describing elastodynamic behaviour of the plate structure in plane-stress state. The consideration will be based on the tolerance averaging technique [9,10]. The general results of the contribution will be illustrated by the analysis of the free vibrations of a structure under consideration.

1. Introduction

Introduce the orthogonal Cartesian coordinate system $Ox^1x^2x^3$ in the physical space occupied by a plate structure under consideration. Let $\Xi = (0, L_1) \times (0, L_2)$ be the midplane (the symmetry plane) of the structure. It is assumed that thickness of the plate h and thickness of the ribs b are small compared to the minimum length dimension of the midplane of the plate, $h, b \ll \min(L_1, L_2)$. At the same time the thicknesses h and b are supposed to be small compared to the width of the stiffened ribs H, $h, b \ll H$ (Figs 1 and 2).

Subsequently it will be assumed that number n of the ribs is very large, $1/n \ll 1$, and the maximum distance l between ribs is very small when compared to L_1. Hence $l = L_1/n$ will be treated as a microstructure length parameter. At the same time, the thickness h of the plate is supposed to be small compared to the microstructure length parameter l, $h \ll l$.

The aim of this contribution is to formulate 2D macroscopic models of dynamic behaviour of the plate under consideration. These models will be referred to as asymptotic and tolerance, respectively. By the 2-dimensional macroscopic model we shall understand mathematical model governed by averaged equations of motion with smooth coefficients and unknown functions dependent on coordinates x^1 and x^2.

The formulation of averaged mathematical models of the considered plane structure will be based on the tolerance averaging technique. The general modelling procedures of this technique are given by Woźniak et. al. in books [9,10]. Some applications of the tolerance averaging technique for the modelling of various dynamic problems for elastic micro-heterogeneous structures are given in a series of papers by

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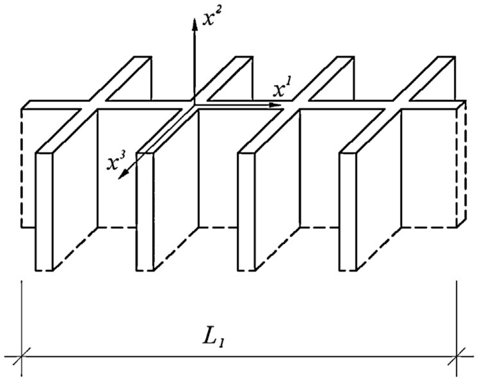

Fig. 1 – Fragment of a plate structure with periodic system of stiffeners.

Baron [1], Jędrysiak [2], Michalak [3], Michalak and Wirowski [4], Nagórko and Woźniak [5], Tomczyk [6], Wągrowska and Woźniak [7], and Wierzbicki and Woźniak [8].

Throughout the paper, indices $i, k, l, \ldots$ run over 1, 2, and 3, indices $\alpha, \beta, \gamma, \ldots$ run over 1, 2 and t stand for the time coordinate. Subsequently we shall use denotations $x = x^1$, $\partial_1 = \partial/\partial x^1$, $\partial_2 = \partial/\partial x^2$. The summation convention holds all aforementioned sub- and superscripts.

2. Formulation of the modelling problem

The considerations will be based on the well-known equations for the plane-stress state in the plate structure. It is assumed that the undeformed midplane of the plate occupies region $\Xi = (0, L_1) \times (0, L_2)$. Denoting by l distance between the ribs of the plate structure, every Δ_i, where $x_i = l/2 + (i-1)l$, $i = 1, 2, \ldots, n$, $(1/n \ll 1)$, will be referred to the cell in Ξ with centre at x_i (Fig. 3). Let $\overline{\Omega} = \cup \Delta_i \times [0, L_2]$ be region in the physical space occupied by plate structure and $\mathrm{int}(\cup \Delta_i)$-cross section of Ω by every $x^2 \in (0, L_2)$-plane. Let subcells Δ_i^P, Δ_i^S, and Δ_i^{SP} be parts of every cell $\Delta_i(x)$; belonging to plate, ribs-stiffeners and part belonging both to plate and stiffeners, respectively.

The model equations for the dynamic behaviour of the plate structure under consideration will be obtained for plane-stress state in the plate.

Subcells Δ_i^P. Plane stress in plane Ox^1x^2, $n^{33} = 0$, hence

$$n^{11} = \frac{hE}{1-\nu^2}(e_{11} + \nu e_{22}), \quad n^{22} = \frac{hE}{1-\nu^2}(e_{22} + \nu e_{11}),$$
$$n^{12} = \frac{hE}{1+\nu} e_{12}, \tag{1}$$

where $e_{\alpha\beta}$ is strain tensors.

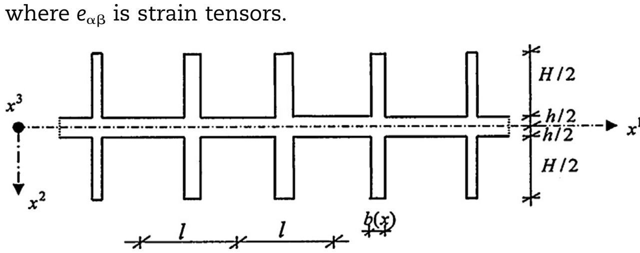

Fig. 2 – Fragment of a cross-section of the stiffened plate structure.

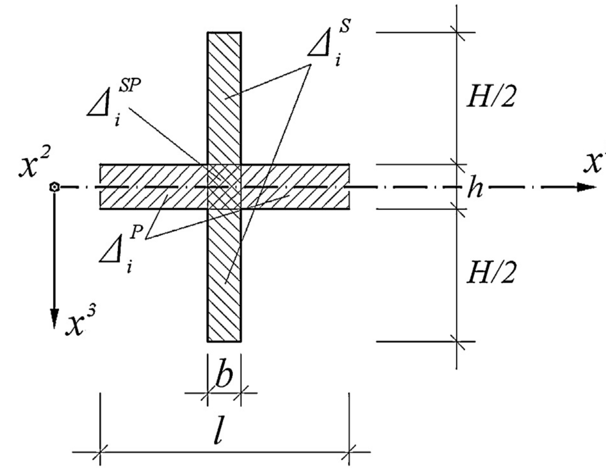

Fig. 3 – The basic cell of the stiffened plate structure.

Subcells Δ_i^{SP}. In this region of the structure we consider 3D-stress state

$$n^{11} = h(\lambda + 2\mu)e_{11} + h\lambda(e_{22} + e_{33}), \quad n^{22} = h(\lambda + 2\mu)e_{22} + h\lambda(e_{11} + e_{33}),$$
$$n^{33} = h(\lambda + 2\mu)e_{33} + h\lambda(e_{11} + e_{22}), \quad n^{12} = \frac{hE}{1+\nu} e_{12}, \tag{2}$$

where λ and μ will be Lame's constants.

Subcells Δ_i^S. Plane stress in plane Ox^2x^3, $n^{11} = 0$, hence

$$n^{22} = \frac{hE}{1-\nu^2}(e_{22} + \nu e_{33}), \quad n^{33} = \frac{hE}{1-\nu^2}(e_{33} + \nu e_{22}), \quad n^{12} = \frac{hE}{1+\nu} e_{12}. \tag{3}$$

Bearing in mind that $h \ll H \ll L_2$ we shall assume approximation $e_{33} \cong -\nu e_{22}$ in subcell Δ_i^S and $e_{33} \cong 0$ in subcell Δ_i^{SP}.

Averaging formulae (2), (3) in Ξ_S (Fig. 4) over $(-(h+H)/2, (h+H)/2)$, with above assumptions, we have

$$N^{11} = h(\lambda + 2\mu)e_{11} + h\lambda e_{22}, \quad N^{22} = [HE + h(\lambda + 2\mu)]e_{22} + h\lambda e_{11}, \quad N^{12} = \frac{hE}{1+\nu} e_{12}, \tag{4}$$

and in Ξ_P averaging formulae (1) over $(-h, h)$

$$n^{11} = \frac{hE}{1-\nu^2}(e_{11} + \nu e_{22}), \quad n^{22} = \frac{hE}{1-\nu^2}(e_{22} + \nu e_{11}),$$
$$n^{12} = \frac{hE}{1+\nu} e_{12}, \tag{5}$$

we derive constitutive equations for 2-dimensional model of the heterogeneous structure under consideration.

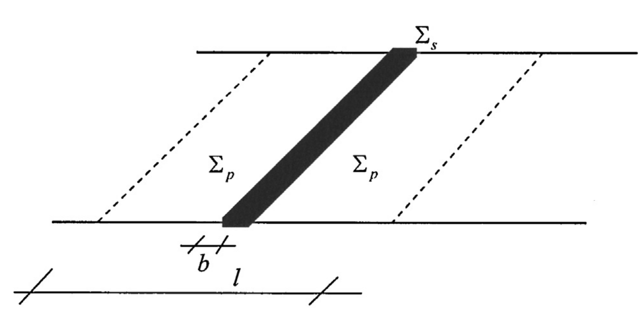

Fig. 4 – Midplane of the plate structure.