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Structural stability and reliability of the underground steel tanks with the Stochastic Finite Element Method

M. Kamiński*, P. Świta

Department of Structural Mechanics, Faculty of Civil Engineering, Architecture and Environmental Engineering, Technical University of Łódź, Al. Politechniki 6, 90-924 Łódź, Poland

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ABSTRACT

The main aim is to present the Stochastic perturbation-based Finite Element Method analysis of the stability and also reliability of the underground steel vertical cylindrical structure of the waste container. This thin walled structure with constant cross-sectional thickness is loaded with subsoil pressure, snow and surface as well as dead loads and we look for the critical pressure value, when stability loss would be observed; it is done to design in the probabilistic context the safety margins. We employ to achieve this goal the Finite Element Method program ROBOT and computer algebra system MAPLE to get the analytical polynomial functions relating the critical pressure and random design parameters – shell thickness and its Young's modulus as well as to provide all probabilistic calculations. We determine up to the fourth order probabilistic characteristics of the structural response assuming that the input random parameters have Gaussian probability functions truncated to the positive values only. Finally, the reliability index is calculated according to the first order method using a difference in-between critical pressure and maximum tensile stress determined in this structure to verify its durability according to the demands of EU engineering codes.

1. Introduction

An uncertainty in geotechnical conditions, external and extra loadings (especially from the moving technical vehicles on the ground surface), as well as material and geometrical imperfections [12] may lead frequently to the catastrophic failure of the underground structures. It can be of the paramount importance in case of the thin-walled shell structures of the underground containers made of the polymers, steel or aluminum, where we may notice the loss of both local of global stability; quite similar phenomenon may be observed in the water or oil pipelines during some technological lack of the internal pressure. Contrary to any structures made of other materials, these made of steel and aluminum need each time precise verification of the stability as the critical forces and moments dominate the engineering designing process and it is checked numerically even for the beams [6]. It significantly

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* *Corresponding author*. Tel.: +48 669001636.
E-mail addresses: Marcin.Kaminski@p.lodz.pl, mm_kaminski@wp.pl (M. Kamiński).

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complicates engineering analysis as theoretical models of the shells stability (stability limits) under such a complex system of external forces and pressures are still being developed, especially accounting for an application of the (Stochastic) Finite Element Method (FEM). The least recognized area here is fully nonlinear stability loss of the steel shells above the yield stress, however stability analysis even in the deterministic context is full of the research challenges that are still open [5]. Buckling of the underground shell may also happen in case of the significant rainfalls or a flood, where the subsoil with large volumetric amount of the water shows extra pressure on the surface of the empty container. This uncertainty leads to a necessity of an application of some stochastic methods of reliability evaluation for such containers, tanks or pipes. We need to include this uncertainty into the computational model provided by some FEM program, so that usually one of the Stochastic Finite Element Methods (Monte-Carlo, polynomial chaos or stochastic perturbation-based) is employed [4]. An application of the SFEM in the area of steel structures analysis has been proposed by Waarts and Vrouwenvelder [15]; its application to the elastic stability analysis could be found in Ref. [8]. The second issue is reliability index calculation, where we need to specify the limit function, whose first two probabilistic moments (expectation and standard deviation) return its value (according to the First or Second Order Reliability Method – FORM or SORM, respectively). The most natural approach in this context for a reliability analysis is a difference in-between critical force or pressure and the corresponding maximum tensile pressure or force acting upon the given structure.

Considering above, we study the critical pressure for the cylindrical vertical underground steel container with Gaussian uncertainty in its cross-sectional thickness and Young's modulus with the use of the Stochastic perturbation-based Finite Element Method [9]. It is implemented in two different computer programs – with the use of the symbolic algebra system MAPLE as well as thanks to the FEM system used in civil engineering – ROBOT [10]. Firstly, the polynomial response functions in-between thickness and Young's modulus and a critical pressure are found through the series with FEM experiments with varying design parameter value. The first computer system is totally responsible for the Weighted Least Squares Method approximation of these analytical functions. A choice of the polynomial order is automatic right now for the first time and this order corresponds to minimum of the mean square error and fitting variance of the WLSM procedure itself. Having optimum responses, we apply stochastic perturbation technique of up to the tenth order to calculate expected values, coefficients of variation, skewness and kurtosis of the critical pressure with respect to the input coefficient of variation fluctuations to verify whether the Gaussian character of the random input is preserved by the linearized buckling problem or not. As the alternative method some semi-analytical technique is proposed, where the same response functions are integrated directly according to the theoretically sound definitions of probability theory for the same variability of the input coefficient of variation.

It should be emphasized that an application of the perturbation-based FEM includes determination of partial derivatives of the structural output with respect to the uncertainty source. So that, it really consists of structural sensitivity analysis – the first order partial derivatives after usual normalization return the additional sensitivity coefficients. They enable here to validate an importance of the particular parameters into a design procedure and they are all computed together with all the probabilistic coefficients, too.

Finally, we calculate the reliability indices for these two randomness sources independently to verify their values according to the demands expressed in the obligatory EU engineering codes. It should be mentioned that the computer power engaged to this study is acceptable for a small scale notebooks, so that may be made ad hoc in situ, almost during the civil engineering inspection. The results obtained in this paper may be directly used to design the extra loads and extra subsoil deposits above the existing containers (together with their coefficients of variation), which is really an important issue in modern civil and environmental engineering.

2. Probabilistic analysis

Let us consider random parameter b and its probability density function as $p_b(x)$. We use Gaussian variable truncated to the physically admissible values like positive for the Young's modulus etc., where the expected values and its central mth central probabilistic moments are defined as [2]

$$E[b] \equiv b^0 = \int_{-\infty}^{+\infty} b \; p_b(x) \mathrm{d}x, \tag{1}$$

and

$$\mu_m(b) = \int_{-\infty}^{+\infty} (b - E[b])^m \, p_b(x) \mathrm{d}x \tag{2}$$

The major idea of this method is to expand all the input variables via Taylor series about the additional expected values using perturbation parameter ε. The random critical force P_{cr} depending on some random input quantity b is calculated from its definition as [9]:

$$P_{cr} = P_{cr}^0 + \sum_{n=1}^{N} \frac{1}{n!} \varepsilon^n \frac{\partial^n P_{cr}}{\partial b^n} (\Delta b)^n, \tag{3}$$

where

$$\varepsilon \Delta b = \varepsilon (b - b^0) \tag{4}$$

is the first variation of b around its expected value b^0 [11] and where N is arbitrarily chosen even perturbation expansion limit established after several computational verifications that guarantee satisfactory probabilistic convergence of the analysis. Let us consider the random critical force P_{cr} and let us calculate the expected value of by expanding according to formula (3) and inserting into the definition (1). There holds

$$\begin{aligned} E[P_{cr}] &= \int_{-\infty}^{+\infty} P_{cr}(b) \, p_b(x) \mathrm{d}x \\ &= \int_{-\infty}^{+\infty} \left(P_{cr}^0 + \sum_{n=1}^{N} \frac{1}{n!} \varepsilon^n \frac{\partial^n P_{cr}}{\partial b^n} \Delta b^n \right) p_b(x) \mathrm{d}x. \end{aligned} \tag{5}$$

This power expansion in case of the Gaussian distribution is significantly reduced to the even order terms (as the odd