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# Mechanics of adhesive joints as a plane problem of the theory of elasticity. Part II: Displacement formulation for orthotropic adherends

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ABSTRACT

The present paper is a continuation of the general formulation discussed in the paper: Mechanics of adhesive joints as a plane problem of the theory of elasticity. Part I: general formulation, Archives of Civil and Mechanical Engineering 10 (2) (2010). Adhesive joints between adherends of varying thickness, made of various orthotropic materials are considered. The adhesive surface can be curved and shapes of the adherends are arbitrary. The joints can be loaded with shear stress distributed arbitrarily on the adherend surfaces as well as by normal and shear stresses of any distribution along the adherend edges. The problem is formulated in terms of displacements in the form of four partial differential equations of the second order. The boundary conditions allow for existence of sharp edges of adherends. Notions of the obtuse sharp edge and the tangent sharp edge are introduced. The presented examples include complete solutions within the framework of the plane theory of elasticity for adhesive joints with adherends of constant thickness and adherends of varying thickness and curved adhesive surface. The influence of sharp edges on the reduction of stress concentration in the adhesive and the adherends is illustrated. The assumptions, notation and equations presented in the general formulation of Part I are used.



## 1. Introduction

The subject of the first part of this research [1] is an analysis of adhesive joints consisting of two thin sheet adherends with an adhesive in between. It is assumed, that the adherends are thin and have constant or gently varying thickness. The middle surfaces of adherends can be plane or slightly curved. The adhesive between the adherends is thin and can have constant or gently varying thickness. Its middle surface can be plane or gently curved. An adherend is considered as thin if the ratio of its thickness to its dimension parallel to a direction of loading action is less or equal to 0.1. A gentle variation of the adherend thickness means, that absolute values of the first derivatives of the functions describing the thickness of the adherend do not exceed 0.2. The thickness of the adhesive is measured in the direction normal to its middle surface.

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The adhesive joint is modelled as a plane two-dimensional system parallel to the plane 0XY in the orthogonal system of co-ordinates 0XYZ. Projections of the thin sheet adherends and the adhesive on the plane 0XY form identical figures of an arbitrary shape. Loading of the adhesive joint can consist of forces parallel to the plane 0XY distributed on surfaces and edges of the adherends (Fig. 1).

It is assumed, that effects of bending and torsion in thin sheet adherends are of secondary importance and can be neglected in the analysis. The thickness of the adherend is measured in the direction normal to the plane 0XY. Due to this and to the assumption, that the adherends are thin and have constant or gently varying thickness it is further assumed, that stresses across the adherend thickness are constant and form a plane stress state parallel to the plane 0XY. The thin sheet adherends of the joint are considered as plane elements parallel to the plane 0XY. The thickness of the adherends 1 and 2 is described by functions $g_1 = g_1(x, y)$ and $g_2 = g_2(x, y)$. It is assumed, that they are $C^1$-continuous with respect to the variables $x$, $y$. The functions $g_1$ and $g_2$ can be zero on some sections or in a neighbourhood of certain points on the adherends edges. The adhesive thickness function $t = t(x, y)$ is positive and is $C^1$-continuous with respect to the variables $x$, $y$.

The adhesive is modelled as an isotropic linearly-elastic medium with the Young's modulus $E_s$, the shear modulus $G_s$ and the Poisson's ratio $\nu_s$, where $E_s = 2(1 + \nu_s)G_s$. Stresses in the adhesive are defined as interactions between the adherend 1 and the adhesive. There are shear stresses $\tau_x = \tau_x(x, y)$, $\tau_y = \tau_y(x, y)$ tangent to the adhesive middle surface and a stress $\sigma_N = \sigma_N(x, y)$ normal to the adhesive middle surface. The stress $\tau_x$ acts along the tangent parallel to the plane 0YZ and the stress $\tau_y$ – along the tangent parallel to the plane 0XZ. The shear stresses $\tau_x$ and $\tau_y$ in the adhesive are positive when their projections on the plane 0XY coincide with the positive orientation of the axes X and Y. The tensile normal stress $\sigma_N$ is assumed to be positive. It is assumed, that the stresses in the adhesive are constant across its thickness. As a result of action of the shear stresses $\tau_x$ and $\tau_y$ in the adhesive, shear strains occur. They evoke relative displacements of adhesive layers in the directions tangent to the adhesive middle surface. The stress $\sigma_N$ evokes strains in the adhesive, which are normal to the adhesive middle surface.

From the assumptions, that the adhesive joint is loaded by forces parallel to the plane 0XY and that the adherends are subjected to the plane stress state parallel to the plane 0XY it results, that a resultant from the stresses $\tau_x$, $\tau_y$ and $\sigma_N$ is parallel to the plane 0XY.

Displacements of the adherends 1 and 2 are described by functions $u_1 = u_1(x, y)$ and $u_2 = u_2(x, y)$ for the direction X and functions $\upsilon_1 = \upsilon_1(x, y)$ and $\upsilon_2 = \upsilon_2(x, y)$ for the direction Y. It is assumed, that the functions $u_1$, $u_2$, $\upsilon_1$ and $\upsilon_2$ are $C^2$-continuous with respect to the variables $x$, $y$.

A uniformly distributed loading on external surfaces of the adherends 1 and 2 expressed in terms of components parallel to the axes X and Y is denoted by $q_{1x} = q_{1x}(x, y)$, $q_{2x} = q_{2x}(x, y)$ and $q_{1y} = q_{1y}(x, y)$, $q_{2y} = q_{2y}(x, y)$. The loading components are positive if they coincide with the orientation of the axes X, Y.

The present paper is a continuation of that general formulation from Part I [1]. An adhesive joint between adherends made of orthotropic materials is considered. The problem is formulated in displacements and a set of four partial differential equations of the second order is derived. Boundary conditions allow for existence of sharp edges of adherends. The notions of *obtuse* and *tangent sharp edges* were introduced. The equations were solved using the finite difference method. The presented examples include complete solutions within the frame of plane theory of elasticity for adhesive joints between adherends of constant thickness as well as varying thickness and curved adhesive surface. The influence of sharp edges on the reduction of stress concentrations in the adhesive and the adherends is illustrated.

The assumptions, notation and equations presented in the general formulation of Part I [1] are used.

All the presented examples should be considered as illustrative.

## 2. Displacement equations for joints between orthotropic adherends of varying thickness and curved adhesive surface

It is assumed that the adherends are generally made of different orthotropic materials, and that the main axes of orthotropy coincide with the axes X and Y. The constitutive equations for adherends take the form

$$\varepsilon_{kx} = \frac{1}{E_{kx}}\sigma_{kx} - \frac{\nu_{kxy}}{E_{ky}}\sigma_{ky}, \tag{1.1}$$

$$\varepsilon_{ky} = -\frac{\nu_{kyx}}{E_{kx}}\sigma_{kx} + \frac{1}{E_{ky}}\sigma_{ky}, \tag{1.2}$$

$$\gamma_{kxy} = \frac{1}{G_{kxy}}\tau_{kxy}, \tag{1.3}$$

where $k = 1$ for the adherend 1 and $k = 2$ for the adherend 2.

An orthotropic material in the plane stress state is characterised by five material constants: two longitudinal stiffness moduli $E_{kx}$, $E_{ky}$, one shear stiffness modulus $G_{kxy}$ and two Poisson's ratios $\nu_{kxy}$, $\nu_{kyx}$. It is assumed that the conditions

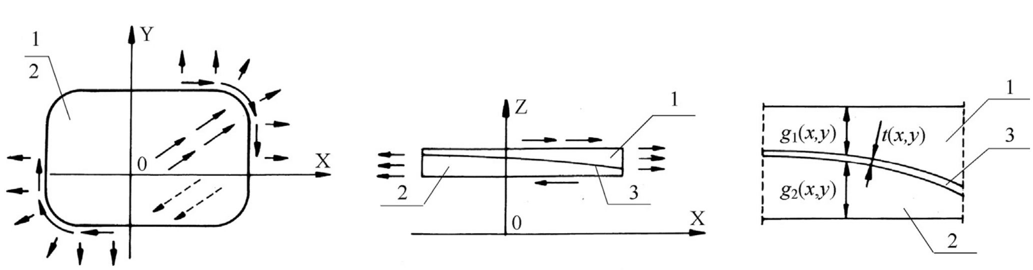


**Fig. 1 – Layout of an adhesive joint. 1 - adherend 1, 2 - adherend 2, 3 - adhesive.**