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Original Research Article

Method of identifying nonlinear characteristic of energy dissipation in dynamic systems with one degree of freedom

M. Bocian*, M. Kulisiewicz

Wroclaw University of Technology, Institute of Materials Science and Applied Mechanics, Smoluchowskiego 25 Str., 50-370 Wroclaw, Poland

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ABSTRACT

A method of determining the shape of the vibration damping characteristic in systems with one degree of freedom in the case when the characteristic depends not only on the velocity but also in an unknown way depends on the displacement has been developed. The method is intended for determining the specific form of the mathematical function describing this dependence. The method utilizes an appropriate analysis of experimentally determined traces of free vibrations of the system. The method has been verified on a few selected computer systems.

1. Introduction

The dynamic properties of most construction materials are usually described by a stiffness parameter and a viscous damping parameter determining the rate of energy dissipation. The parameters stem from the traditionally adopted rheological Kelvin model [1], which was the basis for developing, among other things, the well-known method of analyzing dynamic mechanical systems, called experimental modal analysis [2–4]. However, the analytical description of the dynamic properties of novel construction materials as well as biological materials (biomaterials) often poses difficulties due to the fact that the *strain–stress* dependence in these materials is not linear. This is observed already for quasi-static loads inducing deformations with given constant velocities. Moreover, the *strain–stress* dependences obtained at constant velocities often depend on the velocity with which specific strength tests (tensile, compressive, torsional, etc.) are conducted while the moduli of elasticity determined in this way often significantly depend on the rate of deformation and change depending on the deformation level (nonlinear dependence). This is observed not only in the case of many non-metallic construction materials (plastics and composites), but also for the deformation of biological materials, e.g. human and animal bones [5–9]. This indicates that the linear Hooke model which is traditionally adopted in the mechanics of materials is unreliable for a very wide range of materials.

* *Corresponding author*. Tel.: +48 71 320 27 54.
E-mail address: miroslaw.bocian@pwr.wroc.pl (M. Bocian).

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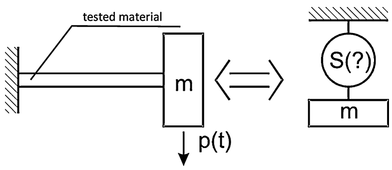

Fig. 1 – Example of dynamic system in which tested material acts on concentrated mass *m* in accordance with Eq. (1).

Hence there is a need to look for new rheological models of materials and to develop new identification methods for them.

The idea put forward by the authors of this paper is to build such a (possibly simplest) dynamic system whose elasto-damping element would be wholly made of the tested material while the movement of concentrated mass m attached in a specified point to this element would be described by a simple differential equation in the form

$$m\ddot{x} + s(?) = p(t) \tag{1}$$

where x stands for time function $x(t)$ describing the movement of mass m, and $p(t)$ is the exciting force. Moreover, it is assumed that the action of the tested elasto-damping element on mass m is described by unknown force S which is a function of displacement x and velocity v. An example of such solution can be a beam made of the tested material that is rigidly constrained at one end and a concentrated mass m is attached to the other end (Fig. 1). If mass m jest is sufficiently large in comparison with the mass of the beam, then one can assume that Eq. (1) sufficiently accurately describes the vibrations of mass m. In this paper it is assumed, a priori, that force $S(x,v)$ has the form

$$S(x, v) = [k + \kappa(x)]v + f_s(x) \tag{2}$$

where $\kappa(x)$ is a certain unknown displacement function satisfying the condition

$$\kappa(x = 0) = 0 \tag{3}$$

It is apparent that for $\kappa(x) = 0$ and $f_s(x) = cx$ relation (2) defines force S in the Kelvin model, i.e. the model commonly used in engineering practice to describe the mechanical vibrations of dynamic systems. Term $\kappa(x)$ (in any form) was introduced because damping properties often depend on the level (state) of material deformation. For example, the resistance of the fluids moving in the canaliculi of a deformed bone is greater than in an undeformed bone since the canaliculi in the former bone are narrowed.

To sum up, a dynamic model of a system described by the equation

$$m\ddot{x} + [k + \kappa(x)]\dot{x} + f_s(x) = p(t) \tag{4}$$

was adopted in this paper. A scheme of the model is shown in Fig. 2.

The aim of the method described below is to determine constant k and function $\kappa(x)$ for arbitrarily nonlinear elasticity characteristic $f_s(x)$ describing pure elastic interactions. Eq. (4)

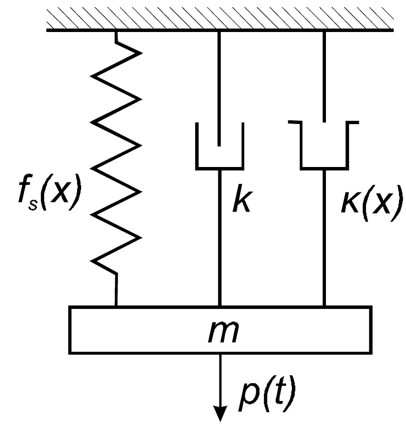

Fig. 2 – Scheme of adopted model.

leads to a qualitatively new process modeling of vibration of mechanical systems similarly as it have place today in mechatronics [10,11].

2. Theoretical basis of the method

Differential equation (4) describes the motion of concentrated mass m in the adopted dynamic model having the form shown in Fig. 2. In the case of nonlinear and unknown functions $\kappa(x)$, $f_s(x)$, it is impossible to obtain solution $x(t)$ of this equation. One can notice, however, that for oscillating motion there exist such instants t_i in which acceleration $\ddot{x}(t)$ is equal to zero. In such instants velocity $v = \dot{x}$, regardless of the form of the solution, must reach extreme values since $\ddot{x} = dv/dt$.

In order to facilitate the analysis let us assume that $p(t) = 0$, which corresponds to free vibrations. Thus for instant $t = t_i$ one gets:

$$\ddot{x}(t_i) = 0, \ \ \dot{x}(t_i) = v(t_i) = v_i, \ \ x(t_i) = x_i \tag{5}$$

According to differential equation (4), values v_i, x_i for $p(t) = 0$ must satisfy the following algebraic equation:

$$[k + \kappa(x_i)]v_i + f_s(x_i) = 0 \tag{6}$$

Hence the following relation is obtained:

$$-\frac{f_s(x_i)}{v_i} = k + \kappa(x_i) = y(x_i) \tag{7}$$

where $y(x)$ is a certain unknown function of variable x. As one can notice, the graph of this function corresponds to that of the characteristic $\kappa(x)$ shifted parallel by constant k.

Relation (7) can be used to determine constant k and function $\kappa(x)$ in a given interval of variable x if:

- a certain set of data x_i, v_i is experimentally determined (by measurement) for values sufficiently densely filling the given interval of variable x,
- function $f_s(x)$ is known (e.g. has been determined by static measurements) in the given interval of variable x.