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A semi-analytical solution on static analysis of circular plate exposed to non-uniform axisymmetric transverse loading resting on Winkler elastic foundation

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ABSTRACT

This paper is concerned with static analysis of functionally graded (FG) circular plates resting on Winkler elastic foundation. The material properties vary across the thickness direction so the power-law distribution is used to describe the constituent components. The differential transforms method (DTM) is utilized to solve the governing differential equations of bending of the thin circular plate under various boundary conditions. By employing this solution method, governing differential equations are transformed into recurrence relations and boundary/regularity conditions are changed into algebraic equations. In this study, the plate is subjected to uniform/non-uniform transverse load in two cases of boundary conditions (clamped and simply-supported). Some numerical examples are presented to show the influence of functionally graded variation, different elastic foundation modulus, and variation of the symmetrical transverse loads on the stress and displacement fields. Based on the results, the obtained out-plane displacement coincide with the available solution for a homogenous circular plate. It can be concluded that the applied method provides accurate results and it is easily used for static analysis of circular plates on an elastic foundation.

1. Introduction

Extensive use of circular plates in particular purposes such as bridge decks, turbine disk, thrust bearing plates and clutches, tanks, structural components for diaphragms, and deck plates in launch vehicles, engineering and spatial structures reflects the importance of circular plates. Since scientists focus on functionally graded material (FGM) to such an extent in engineering field recently, in this paper, FG circular plate is considered. FGMs are new materials, microscopically inhomogeneous continua, where continuous variation of the mechanical properties, from metal to ceramic, happens gradually without any sudden changes. For the first time in an industrial application, Japanese scientists proposed FGM for thermal barriers in aerospace structures [1].This kind of new composites can be found in aerospace structures, nuclear reactors, chemical plants, semiconductors and biomedical

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industries. Comprehensive works on static and dynamic responses of FG plates are available in the literature. Next, we briefly concentrate on some recent works related to the static behavior of FG circular plates.

Reddy et al. [2] investigated the axisymmetric bending of functionally graded circular and annular plates. They studied the bending behavior of plate based on the first order shear deformation Mindlin plate theory. In their study, the Mindlin solution of FG circular plate was obtained for the conditions where the Kirchhoff solution for thin plate was formerly known. Li et al. [3] developed the incremental load technique for solving the governing differential equation of thin circular plate bending with large deformations. In this technique, total applied load was divided into different small steps so that linear stress analysis for the plate was reasonable. Civalek [4] employed differential quadrature method (DQM) and harmonic differential quadrature method (HDQM) in analyzing static and vibration of columns as well as circular and rectangular plates. He compared accuracy of the two methods in structural analysis and showed that HDQM needs less grid points than DQM to achieve accurate results. Li and Ding [5], investigated bending of transversely isotropic circular plates, whose elastic compliance coefficients are arbitrary functions of the thickness coordinate, exposed to a transverse load as a power function of radius. Zheng and Zhong [6] investigated axisymmetric bending problem of FG circular plates under two boundary conditions, rigid slipping and elastically supported, subjected to transverse normal and shear loadings. They utilized Fourier–Bessel series as the displacement function. Civalek and Ersoy [7] studied free vibration and bending of Mindlin circular plates based on the discrete singular convolution method (DSCM) with the use of regularized Shannon's delta kernel. They obtained the frequency parameters, deflections, and bending moments and showed that the singular convolution method is an exact method. Sahraee and Saidi [8] investigated axisymmetric bending of functionally graded circular plates under uniform transverse loadings using the fourth-order shear deformation plate theory. They studied the effect of various percentages of ceramic–metal volume fractions on maximum out-plane displacement and shear stress. Their results were compared with those obtained based on the first-order shear deformation plate theory, the third-order shear deformation plate theory of Reddy and the exact three-dimensional elasticity solution and found good agreement between them. Sahraee et al. [9] analyzed bending and buckling of thick circular FG plate based third-order shear deformation plate theories. They applied the shear–free constraint on the top and bottom of the plate and obtained the static response and critical buckling loads in bending and bucking analysis of functionally graded circular plates using unconstrained shear stress theory in terms of the corresponding quantities of the homogeneous plates based on the classical plate theory. Yun et al. [10] carried out bending analysis of transversely isotropic circular plates under arbitrary symmetric transverse loads. They expanded the transverse loading as Fourier–Bessel series. In their work, the material properties varied arbitrarily along the thickness of the plate. They used the direct displacement method for obtaining the analytical solution. Chen [11] suggested an innovative technique for solving nonlinear differential equations for bending problem of a circular plate. He used a type of pseudo-linearization to obtain the final solution for large deformations of the circular plate. Alipour and Shariat [12] proposed stress analysis for axisymmetric bending of circular FG sandwich plates subjected to transversely distributed loads. They derived the governing equations based on elasticity-equilibrium-based zigzag theory. They employed a semi-analytical Maclaurin-type power-series solution.

In numerous engineering applications, the plate is continuously supported within the span. In the case where the support is linear elastic, its reaction is proportional to the local deflection of the structure (so-called Winkler's elastic foundation). Accordingly, if the plate is supported by an elastic foundation, it experiences a local deflection w, and the reaction (counter-pressure) applied by the foundation to the plate is kw where k is a proportionality coefficient called the modulus of the foundation [13]. In other words, Winkler's elastic foundation is assumed to behave linearly. It should be noted that interaction between plate and elastic foundation is a complicated issue which is not easy to be explored. In many practical engineering applications, this kind of model provides satisfied results. It is worth mentioning that the plates resting on an elastic foundation have been greatly used in modern engineering structures such as building footings, reinforced concrete pavements of high runways, foundation of deep wells, storage tanks, base of machines, aerospace, biomechanics, petrochemical, civil, mechanical, electronic, nuclear and foundation engineering. Providing the exact solution for governing equations of static behavior and dynamic response of any kind of plate in shape under various form of loading is not always feasible. So the researchers attempt to employ the semi-analytical and numerical methods when involved the problems in this field of study. For the first time, differential transformation method (DTM) was introduced by Zhou [14] for solving linear and nonlinear initial value problems in electric circuit analysis. This method is a semi-analytical-numerical technique based on Taylor series expansion developed for various types of differential equations. Differential transforms method solves a series extremely shorter and faster than high order Taylor series method. It also significantly reduces the computation cost of linear and nonlinear problems and is easily applicable. By using DTM, governing differential equations are reduced to the recursive relations together with associated boundary conditions which can be transformed to a set of algebraic equations. Furthermore, this method reduces the computational difficulties of the other methods since all the calculations can be made with a simple iterative process [15]. Another advantage of this method is exact results which can be obtained with a rapid convergence.

DTM has recently attracted the attention of scientists in various fields of engineering. Yalcin et al. [16] represented free vibration analysis of circular plates by differential transformation method. Özdemir and Kaya [17] investigated flap wise bending vibration of a rotating tapered cantilever Bernoulli–Euler beam by differential transforms method. Balkaya and Kaya [18] employed differential transforms method to predict the vibrating behavior of Euler–Bernoulli and Timoshenko beams resting on an elastic foundation (elastic soil). They showed that it is a useful tool for analytical and numerical solutions and that the solution procedure can be easily applied