



# Predicting project progress via estimation of S-curve's key geometric feature values



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## ABSTRACT

The S-curve is a commonly used tool for project planning and control that depicts a construction project's cumulative progress from start to finish. As an alternative approach to estimating S-curves, empirical models derive from progress data of past projects and use mathematical formulas to make progress a function of time. A previous study proposed a cubic polynomial for generalizing S-curves as well as a four-input neural network model for assessing the polynomial's two parameters in order to produce S-curve estimates. This paper presents an improved model, in which the two key geometric feature values of an S-curve, i.e. the position of, and the slope at, its inflection point, are used to replace the polynomial parameters as model outputs. Because these values are likely to be influenced by project conditions, two factors representing project conditions, i.e. degree of project simplicity and degree of team competence, are used as model inputs in addition to the previous four. Data on the nature and actual progress of 51 recently completed projects in the greater Kaohsiung area of Taiwan was collected to illustrate model development, in which the Levenberg–Marquardt algorithm was used to build neural networks for mapping of the input–output relationships. The new model was found to outperform other models in progress prediction accuracy for the project data collected, while sensitivity analysis confirmed its robustness.

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## 1. Introduction

An S-curve depicts the cumulative progress of a construction project from start to finish. The varying slope of an S-curve indicates the changing progress per unit of time that is small at the beginning, becomes larger in the middle, and decreases towards the end. This pattern is typical in construction due to a project comprising multiple activities whose times overlap to some extent, resulting in work relatively concentrated in the middle. The cumulative progress shown by the vertical coordinate at a time point is obtained by multiplying the percent complete of each activity at that time by their respective percent weight and adding up the products. Since every project is a unique undertaking with many influencing factors, every actual S-curve is also unique with different geometric properties.

Using single numbers to represent project progress, an estimated S-curve is easy to comprehend and it has long been widely used for project control by comparison with actual progress, as well as for cash flow planning by combination with optimization methods, e.g. Jiang et al. [6]. Hence, obtaining a reasonable S-curve estimate before a project starts is always of interest to all parties concerned. When detailed information for a project is available, the traditional approach to S-

curve estimation is analytical and based on a schedule of planned activity times. However, because the actual times of a project's activities are subject to many uncertainties, a schedule-based S-curve estimate also involves much uncertainty. Moreover, for early project planning and forecasting at pre-design stages when project definition is vague, it is difficult to use the traditional approach. As an alternative to estimating S-curves, many empirical models that derive from progress data of past projects and use mathematical formulas to make progress a function of time for simplifying processing have been proposed over the years, e.g. Kenley and Wilson [8], Skitmore [12], Kaka and Price [7], Blyth and Kaka [1], Chao and Chien [2], Chao and Chien [3].

A previous study by Chao and Chien [2] proposed a cubic polynomial for generalizing S-curves as well as a four-input neural network model for assessing the polynomial's two parameters in order to produce S-curve estimates. Comparison with statistical methods shows that the model can achieve error reduction in progress prediction, but its performance can still be enhanced. The present study sought to develop an improved model by changing the outputs to an S-curve's key geometric feature values and adding more inputs relating to project conditions in order to increase the prediction accuracy. In the following sections, the previous work is reviewed first prior to proposing changes in the new model. Based on collected project data, a neural network model using the Levenberg–Marquardt training algorithm is then presented. Accuracy of the model, sensitivity analysis, and application of the model are discussed before conclusions are drawn at the end.

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## 2. S-curve formula and previous model

Chao and Chien [2] proposed the following third degree polynomial function with two parameters  $a, b$  for generalizing S-curves:

$$y = ax^3 + bx^2 + (1-a-b)x \quad (1)$$

where  $y$  and  $x$  denote standardized (percent) cumulative progress and time, respectively.

Eq. (1) can meet an S-curve's boundary conditions of ( $x = 0, y = 0$ ) and ( $x = 1, y = 1$ ) and has a more succinct form than other formulas such as the Logit transformation formula in Kenley and Wilson [8]; hence it is more convenient when used in calculating progress. Setting the values of  $a, b$  properly can produce a suitable S-curve with an inflection point connecting two arcs, a convex one followed by a concave one, within the two boundaries, while changing the values of  $a, b$  can produce S-curves with different geometrical properties. Fitting Eq. (1) to actual project progress data is achieved by using the least squared error method for solving  $a, b$  to obtain an S-curve closest to all data points; the solutions derived are shown below.

$$a = (AB - DE) / (BC - E^2) \quad (2)$$

$$b = (CD - AE) / (BC - E^2) \quad (3)$$

where

$$A = \sum x^3 y - \sum xy - \sum x^4 + \sum x^2 \quad (4)$$

$$B = \sum x^4 - 2 \sum x^3 + \sum x^2 \quad (5)$$

$$C = \sum x^6 - 2 \sum x^4 + \sum x^2 \quad (6)$$

$$D = \sum x^2 y - \sum xy - \sum x^3 + \sum x^2 \quad (7)$$

$$E = \sum x^5 - \sum x^4 - \sum x^3 + \sum x^2. \quad (8)$$

The root of mean squared error (RMSE) is used to measure an S-curve's closeness of fit to actual progress data as well as to evaluate a model's performance in predicting progress, as defined next:

$$RMSE = \sqrt{\frac{\sum_{i=1}^d (\hat{y}_i - y_i)^2}{d}} \quad (9)$$

where  $d$  = number of time points (progress measurements) for a project;  $\hat{y}_i$  = calculated progress from an S-curve formula at time point  $x_i$ ;  $y_i$  = actual progress at  $x_i$ .

Chao and Chien [2] fitted Eq. (1) to the actual progress data of 27 projects in Skitmore [12] as well as 101 projects for the second freeway of Taiwan and measured its closeness of fit. For both sets, an average RMSE of less than 0.025 was achieved, comparable to that of using the widely cited Logit transformation formula in Kenley and Wilson [8]. Chao and Chien [2] also used four project attributes, i.e. contract amount, duration, type of work, and location, as inputs to develop a neural network model for assessing the values of  $a, b$  of Eq. (1) for producing S-curve estimates. The neural network model achieved accuracy in progress prediction for the second freeway projects at an average RMSE of 0.0552, whereas a multiple regression model using the same

attributes achieved that of 0.0586 and the average curve model using none of the attributes achieved that of 0.0609. Presumably, there is a room for improvement in performance of the models.

## 3. Proposed changes in new model

The polynomial S-curve formula in Eq. (1) and the previous model mentioned above serve as the starting point of the present study for developing an improved model. Changes that may be useful for achieving further error reduction in progress prediction are proposed below, which comprise model outputs, model inputs, and the method for model building.

### 3.1. Model outputs

For an S-curve produced from given values of  $a, b$  in Eq. (1), its key geometric features, i.e. the position of its inflection point,  $p$ , and the largest slope of the curve at  $p, s$ , can be derived from  $a, b$ , and vice versa. The first derivative of Eq. (1) with respect to  $x$  is the slope of the S-curve at each time point, which gives project progress per unit of time:

$$y' = 3ax^2 + 2bx + (1-a-b) \quad (10)$$

Proper values of  $a, b$  can make  $y'$  in Eq. (10) increasing, peaking, and decreasing between  $x = 0$  and  $x = 1$ , consistent with a common construction project's progress pattern. The value of  $x$  maximizing Eq. (10) is the position of the S-curve's inflection point. To solve  $p$ , take differentiation of Eq. (10) with respect to  $x$  and set the derivative to zero as:

$$y'' = 6ax + 2b = 0 \quad (11)$$

From Eq. (11), the position of the inflection point of an S-curve ( $p$ ) is solved as:

$$p = x = \frac{-b}{3a} \quad (12)$$

From Eqs. (10) and (12), the slope of the S-curve at  $p$  ( $s$ ) is obtained as:

$$s = y' = \frac{-b^2}{3a} + (1-a-b) \quad (13)$$

In a reverse way,  $a, b$  can be derived from Eqs. (12) and (13) and stated in  $p, s$  as:

$$a = - \left[ \frac{(s-1)}{\left(p + \frac{1}{3p} - 1\right)} \right] \div (3p) \quad (14)$$

$$b = \frac{(s-1)}{\left(p + \frac{1}{3p} - 1\right)} \quad (15)$$

Thus, an S-curve estimate can be made by assessing  $p, s$  first and then translating them into  $a, b$  for producing it using Eq. (1). In order to use  $p, s$  properly, we need to establish the range of their values suitable for an S-curve as the basis of a new model. There are two basic requirements for  $p, s$ : (i)  $0 \leq p \leq 1$  for  $0 \leq x \leq 1$ ; (ii)  $s > 1$ , because if  $s = 1$ , Eq. (1) becomes a straight line,  $y = x$ , with  $a = b = 0$ , and if  $s < 1$ , Eq. (1) gives a curve opposite to the S shape, whose slope at the inflection point is the smallest. Moreover, if the value of  $s$  is too large, an improper S-curve results, in which  $y < 0$  or  $y > 1$  occurs near the start or the end. Hence, there are upper bounds of  $s$ , which depend on  $p$ , e.g. the maximum of  $s$  is about 1.85 for  $p = 0.5$ .

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