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Mathematical formulation and preliminary testing of a spline approximation algorithm for the extraction of road alignments

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ABSTRACT

This paper presents a methodology that allows the identification of the road alignments (curves, straights and clothoids) and their corresponding values of curvature based on a list of points with UTM coordinates obtained from field data. The procedure reconstructs the geometry of a road using a cubic spline that gives the road's singular points, geographically referenced, with the curvature values for each element. The methodology allows the user to select different parameters depending on the road type analyzed. It was applied in almost 1500 km of road with satisfactory results. The results of applying the methodology on a recently built road (whose alignments according to its project are known) are shown. A comparison of the project alignments with the alignments obtained according to the proposed method gives good results with small errors relative values, with maximum values of less than 4%.

1. Introduction

The horizontal geometry of a road is defined as a continuous axis formed by a succession of alignments or elements. Three types of alignments are normally used in road engineering: straight alignments in which the azimuth is constant and the curvature is null (infinite radius); circular curves in which the azimuth varies linearly along the trajectory and the curvature is constant; and transition curves, in which both the azimuth and the curvature vary along the trajectory [1]. The purpose of transition curves is to achieve a gradual transition in the change of direction from a straight to a curved alignment. The transition curve most frequently used for roads is the clothoid. The clothoid is a curve whose curvature changes linearly with its curve length and the product of a curve's radius by the distance to the point where the curvature is null is constant and equal to the clothoid squared parameter.

Many applications used by transportation engineers require the road axis to be defined in terms of alignments, such as, for instance: maintenance and reconstruction works [2]; evaluation of driving conditions and the driver's mental workload [3]; analyses of road visibility [4]; road safety and operating speed; and design consistency [5,6].

A road's alignment is one of the geometric features that have the greatest impact on level of service and safety. Certain accidents frequency studies show that accidents on curved sections of roads are more frequent and severe than accidents on straight sections [7–10]. The determination of alignments is a preliminary step required by the Interactive Highway Safety Design Model [11] and the Highway Safety Manual [12] software (the former is used to analyze design consistency based on speed profiles and the latter provides a method to quantify changes in accident frequency as a function of cross-sectional features).

The alignments of recently constructed roads are easy to obtain from design software. For existing roads, however, the data is not always available, or the format is not appropriate, it is not updated, or it is inaccurate. Significant progress has been made to obtain highway data with geospatial data capture technologies such as GPS (Global Positioning System), airborne and satellite-based remote sensing, map digitalization, and datalog vehicles. Most of these technologies provide a georeferenced set of points.

A vehicle's trajectory or the centerline road axis can be reconstructed relatively easily using this data and this information suffices for many useful applications, such as navigation and route guidance systems. Previous research on centerline geometry extraction from geospatial data capture technologies, concentrates on laborious to fully automated systems that model road axis geometry in the form of a polyline or a line of continuous curvature [13,14].

Some researchers have focused on the use of various types of spline curves due to their flexibility, mathematical simplicity and computational ease [15–17]. A spline is a curve defined by a piecewise, explicit or parametric polynomial function. In approximation problems, polynomial spline functions are often used because they produce good approximation methods using low-grade polynomials and therefore prevent the undesired fluctuations caused by the polynomial approximation. The fact that low-grade polynomials usually perform better than those

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of a high order, because the latter tend to induce high interpolation errors especially toward the data tail ends, is also applied in many engineering problems [18,19]. The most widely used spline functions for smoothness and order of approximation are odd degree splines as opposed to even degree splines and, among the former, cubic splines [17]. Class two cubic splines ensure the existence of a straight section that is tangent to every point on its diagram (class 1, continuous first derivative) and, on the other hand, the continuity of the curvature function all along its domain (class 2, continuous second derivative).

Notwithstanding that splines can adequately describe centerline geometry, situations do exist where road axis geometry should be defined in terms of traditional design elements (i.e., straight lines, circular curves, and clothoids) [2,20]. Dividing the road into alignments depending on its curvature is a far more complicated task. In some works the identification of alignments is done manually, but this is not an accurate procedure and it is exceedingly tedious if used to analyze an entire road network.

Today, the extraction of road alignments usually forms part of broader road inventory projects for which “land mobile mapping systems” are used for field work. Vehicle navigation for such systems relies on the use of integrated Global Positioning System (GPS), Inertial Measurement Unit (IMU) or Distance Measurement Instrument (DMI) through Kalman Filtering. Madeira et al. [21] and Chiang et al. [22], try to improve the Mobile Mapping Systems (MMS) technology used to obtain the road geometry, among other things. Wang et al. [23] recognize that MMS have yielded an enormous time saving in capturing road networks but considered that the manual extraction of the road information from the mobile mapping data is still a time-consuming task. Hence they develop a robust automatic road geometry extraction system developed by Absolute Mapping Solution Inc. (AMS). On the other hand, for railway or tunneling inventory projects, for which typically higher positioning accuracies are required, alternative vehicle navigation systems (such as those based on robotic total stations) may be used [24].

Several authors have proposed different sorts of algorithms to identify alignments based on data from geospatial data capture technologies [2,20,25–29].

Almost all these papers [2,20,25,26,28,29] identified two main phases. In the first stage, the georeferenced set of points is classified into straight lines, circular curves, and clothoids by comparing the angles of consecutive points [25,28,29] or by calculating and comparing the radius of consecutive points [2,20]. The second stage is a curve fitting stage in which the set of points is fitted to mathematical curves based on the analytical definition of the geometric elements (straight lines, circular curves, and clothoids): Jiménez et al. [20] use the minisum objective (minimization of the sum of the absolute value of distances between the set of points and the circumference) when fitting the alignments, whereas other authors [2,25,26,28,29] use a least squares regression approach. While most authors estimate the geometric parameters of alignments individually, contrary to existing methods that estimate the geometric parameters of highway and alignments individually, Tong et al. [29] discuss how to extract the entire set of parameters of combined alignments simultaneously using an integrated estimation method.

In this paper we propose a method based on splines to identify road alignments (straight lines, circular curves, and clothoids), thus overcoming the main limitation that many authors have objected to in this technique [2,20]. Firstly, geospatial data capture technology is used to adjust a spline to the georeferenced set of points on the road. Then the spline is used to determine the curves, straight lines and clothoids. This method enables road alignments to be adjusted with or without clothoids.

This proposal is based on the idea set out by Cafiso and Di Graziano [27]. However, they do not give a detailed description of or validate the methodology they propose, nor do they consider a solution for special cases, such as S curves.

This paper is divided into four sections. Following the Introduction, the proposed methodology is described in Section 2 and a case study in which the method is applied is presented in Section 3. The conclusions and a brief discussion for future research are presented in Section 4.

2. Methodology

2.1. The methodology

A seven step methodology is proposed.

The software used has been Mathematica which is a computer software system for doing mathematics [30].

2.1.1. Step 1. Obtain the coordinates of a set of P points of the road

The goal is to obtain the coordinates of a set of road points, normally uniformly spaced; using any type of geospatial data capture technology. Previously, the data must be pre-processed to filter errors in the data collection and eliminate any significantly outliers. Ben-Arieh et al. [16] develop a method for pre-processing data.

Digital road maps can be made in several ways, by digitizing paper maps, taking aerial photographs or using a datalog vehicle. One of the most common techniques to obtain the road geometry is GPS positioning. In this paper let it be assumed that GPS data are used to obtain the road axis that is to be defined in terms of alignments.

2.1.2. Step 2. Adjustment of the variational cubic spline function s to the set of P points obtained

Odd degree splines have graphical representations smoother than those of even degree. This fact together with the fact that low-grade polynomials usually perform better than those of high degree, led us to use cubical splines. Once we had chosen a cubic spline, class 2 ($n-1$) was chosen to ensure continuous functions, and with continuous first and second derivatives (to ensure the continuity of the curvature function) as well.

A class 2 variational cubic spline s that adjusts to the set of road P points whose coordinates were obtained is constructed. At this point in the process, a smoothing parameter intervenes, whose value determines a curve's degree of smoothing as opposed to the approximation of the road points.

A spline is a piecewise polynomial function typically constructed using low order polynomial functions, jointed at breakpoints with certain smoothness conditions. The breakpoints are defined in this context as knots. Diminishing the distance between knots improves the approximation error until reaching a value after which the error remains constant.

If n is the degree of the spline, in order to ensure the smoothness of the approximation, typically $n-1$ continuity conditions should be fulfilled.

Given a partition of $[a, b]$ in m subintervals,

$$\Delta_m = \{a = t_0 < t_1 < \cdots < t_m = b\}. \tag{1}$$

$S_3(\Delta_m)$ denotes the set of cubic spline functions of degree less than or equal to three and class C^2 ($\dim(S_3(\Delta_m)) = m+3$), i.e., every function $s : [a, b] \to R$ such that

i) $s \in C^2([a, b])$,

ii) $s\big|_{[t_{i-1}, t_i]} \in \mathbb{P}([t_{i-1}, t_i]), \quad \forall i = 1, \ldots, m,$

where $\mathbb{P}([t_{i-1}, t_i])$ is the space of all the restrictions of the polynomial functions of a degree less than or equal to three in the interval $[t_{i-1}, t_i]$ [31–33].

Let $\{B_0^3, \ldots, B_{m+2}^3\}$ the B-spline basis of $S_3(\Delta_m)$.