Automation in Construction 47 (2014) 24–36

Contents lists available at ScienceDirect

Automation in Construction

journal homepage: www.elsevier.com/locate/autcon

A simulation-based method for reliability based design optimization problems with highly nonlinear constraints

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ARTICLE INFO

Article history:
Received 26 June 2013
Received in revised form 4 June 2014
Accepted 15 July 2014
Available online 3 August 2014

Keywords:
Reliability based design optimization
Optimum design
Simulation-based design

ABSTRACT

Reliability-based design optimization (RBDO) is a topic of interest in the design of economical structures. It allows designers to effectively reach a balanced cost-safety configuration in the design of structures. In this study, a simulation-based method is presented for RBDO problems in which the design variables are treated as random variables. The method works by uniformly distributing samples in the design space and employing a feature that allows the designer to obtain the optimum design solution by performing only one simulation run. Moreover, the proposed feature also helps the designer to use the results of aforementioned run to provide multi-level design solutions when the arrangement of the design problem is changed. The robustness and accuracy of the method are examined by solving design problems with highly nonlinear constraints and comparing with the results of common RBDO methods. The results confirm the robustness of the method for highly nonlinear problems with different design arrangements.

1. Introduction

Traditional engineering design optimization is limited to deterministic design optimization (DDO) to reduce cost and improve the performance of structures, which can lead to a design with high failure probability because most engineering structures are subjected to uncertain loads and environmental conditions or have unknown material properties or geometric imperfections [1]. These uncertainties can unexpectedly alter the performance of the designed structure and must be considered in the design, maintenance and retrofit of structures [2,3].

Reliability-based design optimization (RBDO) allows designers to achieve a balance between cost and safety, thus producing designs that are not only economical but also reliable in the presence of uncertainty [4,5]. An RBDO problem with multiple reliability constraints is as follows:

$$\begin{array}{lll} \min = c_0(\theta), & & \\ s.t. & P_{F,j}(\theta) \le P^*_{F,j} & j = 1, \ldots, M, \\ c_l(\theta) \le 0 & & l = 1, \ldots, L, \end{array} \tag{1}$$

where $c_0(\theta)$ is the objective function, $\{c_l(\theta) : l = 1, \ldots, L\}$ are the deterministic constraints of θ, $\{P_{F,j}(\theta) \le P_{F,j}^{*}\ j = 1, \ldots, M\}$ are the reliability constraints, and $P^*_{F,j}$ is target failure probability for the j-th reliability constraint [6]. It is clear that the RBDO, as compared with deterministic optimization, requires an additional algorithm to control the specified limit state functions, which depend on random variables [7].

A straightforward approach for solving RBDO problems is a two-level nested optimization [1]. The outer loop does optimization while every candidate solution is evaluated by the reliability analysis in the inner loop. Such methods are referred to as double-loop or nested approaches [8]. There are two types of challenges with this approach. The first lies in calculating the failure probability because probabilistic constraint can be a highly nonlinear function of the original deterministic one [3,6]. During the first attempt to develop an RBDO model, probabilistic constraints are defined by the means of reliability index approach (RIA) [8]. The well-known RIA is based on FORM that approximates failure probability using reliability index. Tu et al. [9] suggested that RBDO should not be limited to RIA and probabilistic constraints' feasibility that can also be evaluated using the performance measure approach (PMA) in a broader perspective [1]. In this approach, an inverse reliability analysis measures the probabilistic performance at the minimum performance target point (MPTP). According to Youn and Choi [10], PMA would be more stable and efficient than RIA due to its less dependence on probabilistic distribution types of random variables. The second challenge is related to the inherent double-loop structure of the RBDO [3]. For both RIA and PMA in RBDO, the optimizer carries out a feasibility search in the outer loop whereas reliability analysis gives probabilistic feasibility in the inner loop. Therefore, the inner loop is required at every outer loop cycle for the reliability analysis [1]. So, despite the conceptual simplicity, these approaches are limited in practical applications since they require too many evaluations of the performance functions [4,6,11].

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http://dx.doi.org/10.1016/j.autcon.2014.07.004
0926-5805/

An alternative to double-loop approaches consists of decoupling the optimization loop from the reliability analysis so that both can be sequentially and independently performed. Such approaches are referred to as decoupled approaches [12–15]. They first determine the deterministic optimal solution in the search space using nonlinear constrained optimizers. Then they find the most probable failure point (MPP) for each constraint. In the next step, constraints are shifted according to their individual MPP. Thereafter, a deterministic optimization to the shifted problem is solved again to finish the current cycle. The entire process continues from cycle to cycle until the convergence [16]. An advantage of decoupled approaches is that they do not require reliability sensitivity analysis because the optimization is performed directly to the performance function [12]. However, decoupling often relies on the MPP assumptions and thus may suffer from possible non-uniqueness of the MPP and strong nonlinearities in the performance functions of probabilistic constraints [4,12,17]. In an attempt to fully reformulate the original RBDO problem into an equivalent deterministic design optimization problem, single-loop methods such as the single-loop single-vector (SLSV) method [18], the single-loop approach (SLA) [19] and other various methods [19,20] replace the probabilistic constraint with an equivalent deterministic constraint by shifting the boundary concept [18]. These methods are very efficient for problems with linear and moderate nonlinear limit state functions.

Recently, a dimension reduction method (DRM) has been developed to approximate the multi-dimensional integration by a function with reduced dimension [21–24]. It was shown in [21] that the DRM-based inverse reliability analysis method can estimate the failure probability of the performance function more accurately than FORM and more efficiently than SORM. This method was successfully applied in RBDO problems, and the accuracy of the solutions was more reliable than traditional FORM-based RBDO methods. The use of meta-models in this field has been proposed in a number of studies [25–27]. Meta-models such as neural networks are used to approximate the relationship between response and design variables in local and/or whole regions of interest [27]. For implicit limit state functions or when performance function evaluation might involve a time-consuming computational task, meta-models replace these functions with those that are easier and faster to evaluate. However, this approach is not always accurate, especially when the objective function and constraints are highly nonlinear [27]. Recent developments in different fields have allowed for the application of the RBDO in a number of challenging problems [16]. For example, the application of advanced simulation techniques allows for the estimation of the reliability of involved structural systems [4,12, 28–30], and stochastic search algorithms [4,31,32] provide the means for efficiently solving RBDO problems. A complete review of the literature concerning the RBDO methods is given in [33,34].

Each of the presented RBDO methods involves advantages and disadvantages. For example, simulation-based RBDO methods are applicable to general nonlinear systems in constraints feasibility evaluations; however, these methods may not be the most efficient methods for special RBDO cases such as linear systems with normal random variables. For such cases, applying approximation-based RBDO methods may lead to an accurate solution with fewer computations. In contrast, when an RBDO approach uses approximation methods in the reliability analysis of implicit constraints with non-normal random variables, the solution may have limited accuracy because the final design solution may not properly satisfy the probabilistic constraints. This can potentially occur with highly nonlinear limit state functions and may lead to an unanticipated decrease in the safety level, which is very dangerous for important structures. Therefore, a proper RBDO design of large nonlinear structural systems should use an efficient and accurate reliability method to feasibly evaluate probabilistic constraints. The design should also use a proper strategy to incorporate reliability analysis into the optimization to decrease the number of computations.

This paper focuses on the special case of RBDO problems in which all design variables of (1) are treated as random variables. For this case, the objective $c_0(\theta)$ may be a function of the random variables parameters. Constraints, however, have no limitations, and the RBDO problem may include both deterministic and probabilistic constraints. In Section 3 of this study, a simple approach is proposed to solve the aforementioned RBDO problems. The method is based on a simulation method that is robust for reliability problems with non-normal random variables and highly nonlinear constraints [35]. It also has the ability to provide multi-level design solutions by using the result of only one simulation run. A review of the used simulation method is presented in Section 2. In this section, an efficient feature of the simulation method that is utilized in the proposed RBDO is also presented. The proposed method and the capabilities of the method to provide multi-level design solutions are presented in Section 3. Five nonlinear RBDO problems with several design configurations are solved in Section 4 to demonstrate the robustness of the method. A review of the advantages and limitations of the method is presented in Section 5. In Section 6, conclusions are drawn.

2. The weighted simulation method

The proposed design method is based on a weighted simulation method that was presented in [35]. This method uniformly generates samples in a random variable space for all random variables and applies a weight index to the samples. The product of the probability distribution functions (PDFs) of the variables is applied to weight the samples. According to this weighting configuration, samples that are located close to the mean have the maximum weights and vice versa. Then, an index function can be defined that separates samples in the failure region ($I_i = 1$) from those in the safe region ($I_i = 0$). Consequently, the probability of failure is

$$P_f = \frac{\sum_{i=1}^{N} I_i.w_i}{\sum_{i=1}^{N} w_i}, \quad \text{where} \quad w_i = \prod_{j=1}^{s} f_j(i) \tag{2}$$

where w_i is the weight of the i-th sample, s is the number of random variables, N is the number of samples and f_j is the PDF of the j-th variable. The method is schematically shown in Fig. 1. The simulation method was presented in [35] with details, and analyses leading to Eq. (2) were presented in [36] and [37].

2.1. Weight flexibility feature of the weighted simulation method

The nature of weighting and sampling in the reviewed simulation method provides the opportunity to compute new failure probabilities by using the results of the first simulation run. For a certain reliability problem, (2) provides accurate failure probabilities by using weight indices and the index function [35]. If the statistical parameters and/or PDF types of the random variables in the reliability problem are changed, the multiple PDF values and, consequently, the attributed weight of each sample must change ($w'_i = \prod_{j=1}^{s} f_j^{New}(i)$). However, the components of the index function (I) remain constant because the locations of the samples do not change.

New weights (w') should be applied to re-compute the failure probability for reformed statistical values of random variables to obtain new results:

$$P^{New}{}_f = \frac{\sum_{i=1}^{N} I_i.w'_i}{\sum_{i=1}^{N} w'_i}, \quad \text{where} \quad w'_i = \prod_{j=1}^{s} f_j^{New}(i) \tag{2 – 1}$$