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Automatic repair of inconsistent pairwise weighting matrices in analytic hierarchy process

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ABSTRACT

The public procurement process involves contractor selection, where AHP (Analytic Hierarchy Process) has been widely applied to reflect decision makers' (DMs) preference and priority on the basis of multiple criteria. Yet, AHP was criticized that DMs often cannot provide strictly consistent comparisons in a pairwise weighting matrix (PWM). The present study proposes an automatic repair procedure for the inconsistent PWM. The procedure seeks a substitute PWM, which can pass the consistency test, and yet being as close as possible to the primitive PWM. The repair is performed by a particle swarm optimization (PSO) algorithm, whose control parameters are tuned using the Taguchi method. The proposed procedure, by automatically adjusting the PWM with the approval of DMs, can greatly facilitate the process of contractor selection and thus expedite the entire procurement function. Two practical cases are used to demonstrate the advantages of the proposed procedure, in comparison with previous work.

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1. Introduction

Public procurement is the activity by which the state, or its delegated authorities and regulated entities acquire goods, services or works through the market. Public procurement makes up an important share of GDP (Gross Domestic Product) in different economics. In Taiwan, public procurement accounts for 10% of the total GDP while it is 15% in the USA and about 16% in the EU [32].

Traditionally, the criterion for contract awarding rests solely on the bidding price, as long as the bid complies with the minimum requirements of specification characteristics. Nevertheless, the lowest bid may not necessarily represent the best value to the government buyer. Thus, there is a tendency to “allow more leeway in analyzing the capabilities of the suppliers and room to award to other than the lower bidder” [18]. While acknowledging this, the government procurement agreement (GPA), concluded under the auspices of the World Trade Organization (WTO), allows the procurement decision to be taken on the basis of the “most advantageous tender” [25]. Non-price factors, such as technical merit or product quality, have been incorporated into the awarding of contracts [29]. Determining the weights of non-price factors in the overall selection entails a framework for group decision making. To that respect, Analytic Hierarchy Process (AHP) has been extensively used around the world [1,3,5,9,14,22].

AHP is a popular method for assisting decision makers in evaluating alternatives with multiple criteria. It was developed in 1970s and has been extensively used since then. The AHP framework provides a comprehensive and rational methodology, which encompasses the following steps: (1) structuring a decision problem in a hierarchy, (2) obtaining the judgment matrix based on pairwise comparisons between alternatives and between criteria, (3) testing consistency until satisfactory, and (4) synthesizing comparisons across various levels to obtain the final weights of alternatives. Users of AHP make judgments on pairwise comparisons according to Saaty's discrete 9-value scale method [27]. The matrix is called a pairwise weighting matrix (PWM).

In addition to the applications in contractor selection, the use of AHP in the field of project management is also widespread: Kamal and Al-Harbi [15] solved the contractor prequalification problem by AHP. Shapira and Goldenberg [30] used AHP to select construction equipment. Lai et al. [16] determined the budget of public building projects with the aid of AHP. De Miranda Mota et al. [7] employed AHP to set the priorities for activities in construction projects. Ahn et al. [2] used AHP to select investment in technological projects within an organization's portfolio.

Despite its wide acceptance, AHP has been criticized on the ground that decision makers (DMs) often cannot provide strictly consistent comparisons [20]. This problem is of a particular concern when the numbers of criteria and alternatives are large. In Saaty's work [28], consistency is verified by the Consistency Ratio (CR) that indicates the probability that the matrix ratings are randomly

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generated. The rule of thumb is that a CR over 0.1 indicates the PWM should be revised.

Subsequent revisions, however, involve locating inconsistent judgments and reassessing the comparisons. The entire process is laborious and tedious as it may need to be repeated many times until satisfactory. Furthermore, when making urgent decisions in public procurement, sorting out inconsistency repeatedly is not only too expensive for busy DMs but also infeasible when the matter cannot be deferred [34].

The purpose of this paper is to propose an automatic repair procedure to seek a substitute matrix, which can pass the consistency test, and yet being as close as possible to the primitive PWM. Note that the proposed procedure does not intend to overwrite the preferences provided by DMs. Instead, by automatically repairing the PWM and letting the DMs approve on the results, the method can greatly facilitate the process of contractor selection and therefore expedite the entire procurement function. Another advantage of the proposed procedure is that it can reduce the high failure rate of AHP caused by using discrete values to represent preferences that should take on continuous values [36]. Since the search space is continuous with no gradient information, the repair is performed by a particle swarm optimization (PSO) algorithm, which has been successfully adapted to solve non-differentiable optimization problems [4, 11]. The proposed procedure is coined as the PSO–AHP procedure. The Taguchi method of experiment design is incorporated into the procedure to reduce the number of experiments required for tuning the control parameters of PSO.

This paper is organized as follows. The next section gives a summary review of AHP. The problem at hand is then formulated as a continuous optimization problem subject to the consistency constraint. Both the PSO algorithm and the Taguchi method are described afterwards. The proposed PSO–AHP procedure is applied in two large scale real-life projects to assist in contractor selection. The practical application results demonstrate the performance of PSO–AHP, whose validity is further confirmed by a comparison with previous work [20]. Conclusions are drawn in the last section.

2. Review of AHP

AHP is a well-known multi-criteria decision making (MCDM) tool. With it, human experts can structure their perceptions hierarchically, compare pairs of elements against a given criterion, and judge the intensity of importance of one element over the other. The ultimate goal is to derive priorities based on sets of pairwise comparisons, which are transformed into a numerical value of the discrete 9-value scale. The verbal interpretations of the scale values can be found in [27].

An MCDM problem consists of multiple levels for decomposition of the decision-making process. Each level has multiple nodes with respect to which the alternatives on its child level are compared. For each level, elements are compared in pairs according to their contribution to the parent node above. The pairwise comparisons are collected in a square matrix $\mathbf{A}=\{a_{ij}, i=1,2, \ldots, n; j=1,2,\ldots n\}$, which is positive reciprocal as the lower triangular matrix is composed of the reciprocal values of the upper triangular while the diagonal is always 1:

$$a_{ij}=\frac{1}{a_{ji}}\,\forall i\neq j \qquad a_{ii}=1. \tag{1}$$

The next step is to synthesize the judgments in the pairwise weighting matrix (PWM) to get the relative priorities. It can be accomplished either by approximation or by the power method. The approximation approach divides each element of the matrix with the sum of its column to get normalized relative weights while the sum of each column is always 1. Approximation works well for small matrices but may associate with errors for large matrices. By comparison, the power method keeps multiplying the PWM by itself, raising the matrix to large powers. The rows of the resulting matrix are added and then normalized. The process stops when the normalized vector of the current power is reasonably close to the previous one. Using either method, one can average across the rows to obtain the normalized principal eigenvector, which is also called the priority vector. This is done for all groups on all levels.

With the priority vectors, two adjacent levels are aggregated using the multiplication of weight values of the child level with the associated weight value of the parent level. In this way, the contribution of a considered criterion in the child level is the accumulative multiplication of all weights along the path from the top criterion to the considered criterion.

A PWM is said to be perfectly consistent if all the transitivity relationships are satisfied

$$a_{ij}=a_{ik}\cdot a_{kj}. \tag{2}$$

Due to the limit of the 9-value scale, the relationships in Eq. (2) are often violated. This is because when two elements are constrained between 1 and 9, their multiplication is very likely to go beyond 9. Therefore, perfect consistency is difficult to achieve. The consistency of pairwise comparisons is measured by the principle eigenvalue, which satisfies the following condition

$$\mathbf{A}\times\mathbf{W}=\lambda_{max}\times\mathbf{W} \tag{3}$$

where λ_{max} is the principle eigenvalue of $\mathbf{A}$ and $\mathbf{W}=(w_1, w_2, \ldots\ldots, w_n)^T$ being the vector of relative weights. The principle eigenvalue may be obtained by the aforementioned power method.

Since a perfectly consistent matrix would have λ_{max} equal to the number of elements being compared, the consistency index (CI) is defined to be

$$\mathrm{CI}=(\lambda_{max}-n)/(n-1) \tag{4}$$

and the consistency ratio (CR) is

$$\mathrm{CR}=\mathrm{CI}/\mathrm{RI} \tag{5}$$

where RI (random index) is the average index of randomly generated weights. It was suggested that the allowable threshold for CR should be no greater than 10%. This is to ensure only one order of magnitude is spanned for perturbations. It has also been realized that the threshold of CR cannot be made too small because it would then prohibit new understandings [28].

3. Problem statement

The most advantageous bid (MAB) method attempts to select a contractor whose proposal is most favorable to the project owner by evaluating not only the bid price but other related factors. Similar multi-criteria selection methods have been adopted in Taiwan, European Union and various countries [8, 19, 38].

Many researchers have identified selection criteria common to various projects [17,26,31,35,39]. In addition to the bidding price, other common criteria can be found in the provisions of Taiwan's Government Procurement Act [23]:

1. Technology: such as functionality of technical specifications, professional manpower and expertise, capability of timely contract performance, technical feasibility, and equipment resources;
2. Quality: such as quality control capability, inspection and testing method, and error detection rate;
3. Function: such as production capacity, robustness, versatility, expandability, compatibility, and adaptability;