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A simplified mathematical approach for modelling stack ventilation in multi-compartment buildings

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ABSTRACT

A simple mathematical model of stack ventilation flows in multi-compartment buildings is developed with a view to providing an intuitive understanding of the physical processes governing the movement of air and heat through naturally ventilated buildings. Rules of thumb for preliminary design can be ascertained from a qualitative examination of the governing equations of flow, which elucidate the relationships between 'core' variables – flow rates, air temperatures, heat inputs and building geometry. The model is applied to an example three-storey office building with an inlet plenum and atrium. An examination of the governing equations of flow is used to predict the behaviour of steady flows and to provide a number of preliminary design suggestions. It is shown that control of ventilation flows must be shared between all ventilation openings within the building in order to minimise the disparity in flow rates between storeys, and ensure adequate fresh air supply rates for all occupants.

1. Introduction

Passive stack ventilation is a popular energy-saving design feature in modern architecture. Significant architectural features such as atria, solar chimneys and double façades are often incorporated into large building designs with a view to assisting stack ventilation and thereby delivering a comfortable internal environment. Such multi-compartment buildings pose a particular design challenge due to the interaction between heat and air flows through different building zones.

Computational tools such as multizone building simulation software [1–4] and computational fluid dynamics (CFD) [5–7] are routinely used to tackle these complex design problems, and are capable of detailed, multi-variable analysis of heat and air flows. Due to their flexibility, however, effective use of these tools requires specialist knowledge to ensure reliability of results – which can vary significantly based on choice of code, grid, domain and user input [8,9] – and carries associated costs in time, labour and computing power.

Simplified mathematical models – which can readily be solved by hand, or numerically with a small computational overhead – therefore still form a crucial part of the design process. Whilst not able to capture the same level of detail as computational tools, these simple models elucidate some of the key relationships between design parameters and thereby provide rapid and intuitive guidance at the preliminary design stage. Indeed, industry standards and design guidance (e.g. Refs. [10,11]) are underpinned by these simple models.

Much existing guidance for stack ventilation focusses on the simple case of a single room, the initial mathematical model for which was developed and validated in small-scale laboratory experiments by Linden et al. [12]. Numerous experimental studies have since extended this work to multi-compartment buildings. Holford and Hunt [13], for example, developed and experimentally validated a simplified mathematical model of stack ventilation flows in a room attached to an atrium; this model was further validated by Ji et al. using CFD [5]. Livermore and Woods [14] experimentally validated a similar model of flows in a two-storey building with a ventilation stack. Chenvidyakarn and Woods [15] also showed that a simple model can capture the behaviour of multiple flow regimes in two interconnected heated spaces. Based on the robustness of these models, the behaviour of stack ventilation flows in multi-compartment buildings and the implications for design have also been investigated in theoretical studies [16–18].

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Nomenclature

Symbol

A	vent area, m^2
A'	effective vent area, m^2
A^*	combined effective vent area, m^2
B	buoyancy flux, m^4 s^{-3}
β	thermal expansion coefficient, K^{-1}
c_d	discharge coefficient, –
c_p	specific heat capacity (of ambient air), J kg^{-1} K^{-1}
C_p	wind pressure coefficient, –
E_g	geopotential energy, J
E_k	kinetic energy, J
g	gravitational acceleration, m s^{-2}
g'	reduced gravity, m s^{-2}
H	zone height, m
ΔH	atrium height above top storey, m
L	number of flow loops, –
N	number of building zones, –
Δp_{crack}	pressure drop across a crack, N m^{-2}
Δp_{energy}	energetic inertia pressure, N m^{-2}
Δp_{stack}	stack pressure, N m^{-2}
Δp_{turn}	pressure drop due to stack turning, N m^{-2}
Δp_{vent}	pressure drop across a vent, N m^{-2}
Δp_{wind}	wind pressure, N m^{-2}
Q	ventilation flow rate, m^3 s^{-1}
$\widehat{Q}$	dimensionless ventilation flow rate, –
R_{A^*}	ratio of vent areas, –
Re	Reynolds number, –
ρ	density of air, kg m^{-3}
S	stack cross-sectional area, m^2
t	time, s
T	temperature, K
u	wind speed, m s^{-1}
V	number of vents, –
$\mathcal{V}$	zone volume, m^3
W	heating rate, W
z	vertical coordinate, m

Subscript

a	atrium
c	ceiling-level
e	external environment
eff	effective
f	floor-level
i	storey number
in	inlet vent
l	flow loop index
m, n	zone index
out	outlet vent
p	plenum
pa	combined value for plenum and atrium
s	storey
tot	total
v	vent index

However, few studies explicitly tackle a general approach to stack ventilation in multi-compartment buildings. Etheridge [19], for example, extends the ‘explicit method’ – a simplified mathematical approach to preliminary design presented in CIBSE guidance [11] – to a number of example multi-compartment buildings. Axley [20] also presents a general ‘loop equation’ method for forming the equations governing ventilation flows in multi-compartment buildings, focussing on applications in multizone software.

In this paper we adapt and build upon this existing work to develop a generalised method for modelling stack ventilation flows in multi-compartment buildings, focussing on the intuitive value of the method for use in preliminary design. In particular, we examine the qualitative relationships between ‘core’ ventilation variables – flow rates, temperatures, heat inputs and building geometry – in order to inform the sizing of ventilation openings. The mathematical model used is deliberately simple; where relevant, we have highlighted how additional detail may be included. This work is intended, firstly, to provide one possible approach for extending existing preliminary design guidance for stack ventilation to multi-compartment buildings; and, secondly, to provide a ‘sense check’ for software modellers with a view to reducing the computational overhead and costs associated with the design process.

In Section 2 we outline the assumptions and approximations used in developing our model; in Section 3 we develop a model for stack ventilation in a general multi-compartment building; and in Section 4 we apply this general model to an example three-storey building with an atrium and inlet plenum, testing some of the qualitative predictions of the model by numerically solving for flow rates in a number of scenarios.

2. Assumptions and approximations

2.1. Variations in density and temperature

Stack ventilation flows are driven by differences in density between internal and external air. These differences in density are typically small such that $(\rho_e - \rho)/\rho_e \ll 1$, where ρ is the density of internal air and ρ_e is the density of external air.[1] Air may then be regarded as incompressible to leading order (Boussinesq approximation [22]) and variations in density and temperature are ignored, except where they appear in driving ‘reduced gravity’ terms,

$$g' = g\frac{\rho_e - \rho}{\rho_e} = g\beta(T - T_e), \tag{1}$$

where T and T_e are the internal and external air temperatures, respectively, $\beta \approx 1/T_e$ is the thermal expansion coefficient of air and g is gravitational acceleration. Since the reduced gravity is a measure of the internal temperature excess, $T - T_e$, we interchangeably use the terminology ‘reduced gravity’ and ‘temperature excess’ throughout.

2.2. Heat transfer

Variations in air density and temperature within a building are generated by heat inputs from occupants, office equipment, solar gains, and so on. This heat is then removed by ventilation, or by transfer into or through the building fabric. For simplicity, we

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[1] For example, when $T = 290$ K and $T_e = 280$ K, $(\rho_e - \rho)/\rho_e = 0.033$ [21].