Building and Environment 49 (2012) 182–192

Contents lists available at SciVerse ScienceDirect

Building and Environment

journal homepage: www.elsevier.com/locate/buildenv

Uncertainty analysis and optimum concentration decay term for air exchange rate measurements: Estimation methods for effective volume and infiltration rate

Hiroyasu Okuyama*, Yoshinori Onishi[1]

Institute of Technology, Shimizu Corporation, 4-17 Etchujima 3-chome, Koto-ku, Tokyo 135-8530, Japan

ARTICLE INFO

Article history:
Received 7 July 2011
Received in revised form
9 September 2011
Accepted 15 September 2011

Keywords:
Air exchange rate
Concentration decay method
Least-squares method
Uncertainty propagation
Effective mixing volume
Discrepancy ratio

ABSTRACT

The widely used concentration decay method for measuring the single-zone air exchange rate has few limitations in its practical application, and several measurement standards employing this method have been published including ISO 12569, ASTM E741-95, and JIS A1406. These conventional methods, however, include several shortcomings that should be improved in terms of data analysis. Therefore, an equation for estimating the air exchange rate by the least-squares method from the number of points on the decay curve of the tracer gas and a method for determining the optimum decay term that minimizes the uncertainty are deduced. The method for calculating the standard deviation of uncertainty is also reconsidered and a new equation is deduced and verified. Furthermore, a method is introduced and verified for evaluating whether the premises of the measurement are sufficiently satisfied, such as the invariability of the air exchange rate and the uniformity of the tracer gas in the zone. In addition, a method for simultaneously estimating the effective mixing volume and the infiltration rate separately would be useful and is deduced. This estimation method's accuracy, which depends on the waveform and frequency of gas emission, is investigated through numerical experiments.

1. Introduction

Building air exchange and infiltration rates are significant factors in many aspects of building performance, including air-conditioning energy consumption, indoor air quality, and condensation. In most cases, however, actual air flow rates are different from the intentions of design and operation because of various factors; consequently, on-site measurement methods remain important technologies.

Direct measurements of air flow rate performed in the immediate vicinity of ventilation installations are insufficient for evaluating actual air exchange rate because shortcuts, bypasses, and leaks in the intake and exhaust routes, not to mention building envelope infiltration, may be present. A measurement method that is better suited to the actual situation is to use the tracer gas concentration to estimate the air exchange rates. There are various air exchange rate measurement methods that employ tracer gas. In general, the indoor air quality will likely differ among the zones within a structure, the application of a multi-zonal air flow rates measurement is preferable, but such methods have not yet been sufficiently disseminated.

In most current cases, doors between zones are left open and mixing fans are used to realize a uniform concentration of tracer gas throughout the building, or, conversely, the doors to the next rooms from the measurement room are sealed to create a single zone. The single-zone concentration decay method is highly practical because it allows for any method of tracer gas emission to be used, requires relatively simple equipment, and can be performed with only a minimal level of data analysis, making it the most widely used method.

Many researchers have been working on the uncertainty or error analysis of air infiltration rate measurements and effective mixing volume estimations. For example, Sherman et al. [1] derived a qualitative error analysis for tracer gas mixing problems, and also examined the question of effective mixing volume. Shaw [2] experimentally compared the effect of tracer gas type on the accuracy of air change measurements. Sandberg and Blomqvist [3] presented a quantitative estimate of the error of decay and constant concentration methods. D'Ottavio and Dietz [4] explored the errors resulting from treating a house as a well-mixed single volume, despite the actual situation being more similar to a two-zone case when a basement is included. D'Ottavio et al. [5] developed mathematical schemes estimating ventilation flows and their associated errors using a multiple perfluorocarbon tracer method. The relevant problems exist also in multi-zonal models that have been examined by many researchers, including the present authors.

* Corresponding author. Tel.: +81 3 3820 6438; fax: +81 3 3820 5955.
E-mail addresses: okuyama@shimz.co.jp (H. Okuyama), ohnishi@shimz.co.jp (Y. Onishi).
[1] Tel.: +81 3 3820 6439; fax: +81 3 3820 5955.

0360-1323/$ – see front matter
doi:10.1016/j.buildenv.2011.09.018

O'Neill and Crawford [6] proposed a recursive least-squares identification algorithm estimating flows and volumes using single tracer gas released by impulse.

In least-squares uncertainty analyses, in order to calculate the expectation of the regression equation error, not only the measurement uncertainty but also the regression equation residual, which includes various other causes of uncertainty, would be utilized. Furthermore, the formulation of uncertainty propagation to the estimated parameters would be also improved.

Almost all tracer gas emission methods described have used decay, square, or impulse emissions. Sinusoidal methods have been insufficiently investigated, however, despite it being well known that any periodic function can be represented by a series of trigonometric functions known as a Fourier series. We therefore investigate the effects of the use of a basic sinusoidal wave.

Even in a single-zone infiltration measurement, to consider the unfavorable influence of tracer gas concentration non-uniformity or to estimate the effective volume, we must consider the single zone using a subdivided multi-zonal model. In a spatially discrete diffusing system model, a finely discretized model with many nodes is generally required for high-frequency excitation, while a rough model with fewer nodes is sufficient for low-frequency excitation. Consequently, we should investigate the relation between the roughness of the model and the period of the sinusoidal excitation.

Furthermore, some measurement methods utilize ordinary differential equations of gas concentration for every short time interval Δt as a regression equation, especially where the time differential term tends to produce high-frequency noise because of, for example, measurement uncertainty. The least-squares method is easily affected by such unfavorable noise, making it necessary to preprocess the time series measurement data by using a low-pass filter.

Measurement uncertainty analysis is important in various fields, making standardization also important. A guide produced by the working group of the Joint Committee for Guides in Metrology (JCGM/WG 1) [7] can be referenced as an example of such standardization. In that paper, however, the parameter estimation, example uses only the arithmetic average, which is likely inferior to the more generic and useful least-squares method.

Several other standards have been published, including ISO 12569 [8] and ASTM E741-95 [9], as well as JIS A1406 in Japan [10]. Each of these standards includes measurement data analysis methods described in an informative annex.

However, these measurement standards share the following points (a)–(d), and the aforementioned measurement methods for simultaneously estimating effective mixing volume and infiltration rate include the following points (d)–(g), each of which require improvement or reconsideration.

(a) *Simultaneous least-squares solutions*
There is an equation for estimating the air exchange rate N from two measurement values on the tracer gas decay curve. But for utilizing more values, those standards describe a graphical method whereby the values are plotted on a logarithmic scale to find a linear fit, and suggest the utilization of some ready-made least-squares computer program.

(b) *Optimum term of concentration decay*
Some of the standards determine an appropriate measurement term by using the ratio of decaying concentration to the initial concentration. However, the basis for doing so is not clarified from the viewpoint of statistical uncertainty analysis. One can presume that an overly long or overly short term will have a deleterious effect due to the concentration measurement uncertainty. In this paper, we will numerically solve the nonlinear optimization problem, thus obtaining a curve that determines the optimum decay term.

(c) *Standard deviation of uncertainty*
The equation for the standard deviation of air exchange rate uncertainty (E_N) under conventional methods and the equation for the standard deviation (σ_N) deduced from the uncertainty propagation equation in this paper have structural differences, and these differences require examination.

(d) *Verification of the fulfillment of measurement prerequisites*
Obtaining accurate estimates of air exchange rates depends on the fulfillment of measurement prerequisites, such as invariability of the air exchange rate and uniformity of the tracer gas concentration. The coefficient of determination (COD) of the least-squares method has been proposed as an evaluation index, but COD is insensitive, as its value can be close to 1 even in cases where prerequisites have not been sufficiently fulfilled. In this paper, therefore, we employ a new index, the discrepancy ratio β, as an improved index of measurement prerequisites [11].

(e) *Method for simultaneously estimating effective mixing volume and infiltration rate*
"Effective mixing volume" is defined in this paper by system identification as a parameter of the differential equation for changes in gas concentration [12]. It can therefore be assumed that this will vary with the waveform and frequency of gas emission, which is an excitation for system identification, and therefore will not necessarily agree with the geometric chamber volume.

(f) *Relationship between periods of the trigonometric series function for gas emission and the effective mixing volume*
Okuyama [12] initially tested a square wave excitation. Square waves can also be expanded as a trigonometric series and have high-frequency terms. The system identification of the single-zone effective mixing volume and the infiltration rate presented in this paper examines such periods by using a pseudo-non-uniform model that represents an uneven distribution of gas concentrations.

(g) *Low-pass filter for preprocessing measurement data*
In the present research the moving term average is tested as a low-pass filter.

2. Estimation and uncertainty evaluation of air exchange rate by the least-squares method

2.1. Regression and solution equations

The initial measurement concentration is written as $C(t_1) = C(0)$, where $t_1 = 0\,(h)$ is the initial time of the concentration decay curve. Elapsed time for the j-th subsequent concentration measurement is written as t_j (h), and the concentration at that time is written as $C(t_j)$. Taking the air exchange rate as N (h^{-1}), the following equation can be derived from the analytical solution for the concentration decay curve:

$$\log_e C(t_j) = -N \cdot t_j + \log_e C(0) \tag{1}$$

The following equation defines the equation error e_j for the least squares. This defines the vector matrix representation y_j, $\mathbf{Z}j$, and $\mathbf{a}$ in the following equation.

$$\begin{aligned} e_j &= \log_e C(t_j) - [-N \cdot t_j + \log_e C(0)] \\ &= y_j - [\,t_j \quad 1\,] \cdot \begin{bmatrix} -N \\ \log_e C(0) \end{bmatrix} = y_j - \mathbf{Z}_j \cdot \mathbf{a} \end{aligned} \tag{2}$$