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Contribution to analytical and numerical study of combined heat and moisture transfers in porous building materials

K. Abahri*, R. Belarbi, A. Trabelsi

LEPTIAB, University of La Rochelle, Av. Michel Crépeau, 17042 cedex1 La Rochelle, France

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ABSTRACT

In this paper, one-dimensional model for evaluating coupled heat and moisture transfer in porous building materials was proposed. The transient partial differential equations system was solved analytically for Dirichlet boundary conditions. It consists first to introduce the Laplace transformation and then to use the potential function technique. This approach allows simplifying the initial mathematical problem to a fourth order ordinary differential equation which can be easily solved. This solution was used to assess the transient temperature and moisture distribution across materials. A comparison with numerical models from Luikov [2] and Vafai *et al.* [12] was performed, a good agreement was obtained.

1. Introduction

The coupled heat and moisture transfer in porous media has been widely studied due to its presence in many fundamental and industrial applications. Particularly in building science, researches have been carried out in the past few decades to improve building energy efficiency and indoor air quality [1].

To describe hygrothermal transfer in capillary porous media, Luikov developed a model [2] assuming analogy between moisture migration and heat transfer. Moreover, he assumed that capillary transport is proportional to moisture and temperature gradients. Also, by analogy with specific heat, he introduced the specific mass capacity which is defined as the derivative of the water content with respect to the mass potential. This model is applicable for both hygroscopic and non-hygroscopic materials. It was used by several researchers [3–7].

In order to solve the coupled system for temperature and moisture potentials, many authors used both analytical and numerical approaches. Generally, the solution of the governing partial differential equations depends on the specific problem considered. Qin *et al.* [8] presented one of these specific cases addressed to a multi-layer building material and they obtained satisfactory numerical and experimental results. Ribeiro [5] presented an exact solution of Luikov system for a moist spherical capillary porous body. Their analytical procedure is a combination of the matrix functions and the Laplace transformation. Chang *et al.* [9] proposed an analytical solution obtained for the slab under natural hygrothermal boundary conditions. Their approach is based on the use of transformation functions. These authors [9] cited the work of Liu *et al.* [4] who introduced potential functions corresponding to their system of equations, by a change of variables for temperature and moisture content. They used the boundary condition of the third kind and compared their results with those of Thomas et al. [10] who conducted experimental studies on wood (pine). In the cited papers [5], there exist analytical difficulties to find the solution when the domain of convergence of the series is limited.

This paper presents a Potential Transfer Function Method applied to partial differential equations. It concerns one-dimensional heat and moisture transfer through a plane geometry of porous building component. Non-symmetric Dirichlet boundary conditions are considered. These conditions are more suitable for applications with high value of the dimensionless Biot number (typically, for cases where the average temperature and relative humidity at the surface of the building component converge very quickly to the ambient temperature and relative humidity as confirmed by Younsi [11]). The inverse Laplace Transformation was exactly obtained. The numerical solution was also performed using the same boundary conditions. Moreover, comparison with other numerical model given by Luikov [2] and that of Vafai *et al.* [12] was achieved.

* Corresponding author.
E-mail address: kamilia.abahri@univ-lr.fr (K. Abahri).

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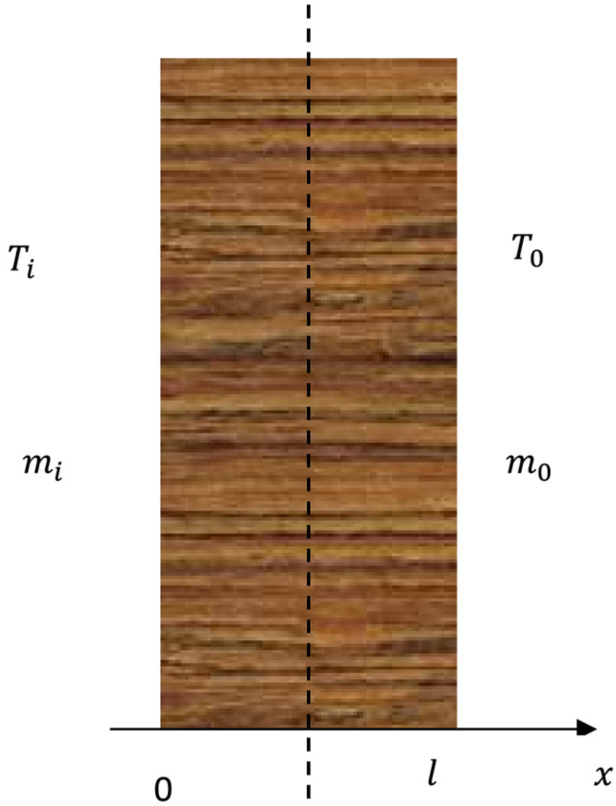

Fig. 1. Schematic representation of the geometry.

2. Analytical modeling of the problem

The used heat and mass transfer model is based on Luikov's [2] approach which is extensively employed in the prediction of heat and moisture migration in porous building materials. However, as usually found in many previous studies, the following assumptions are still adopted in this study:

- The material is considered homogenous and the thermo-physical properties are assumed constant.
- A local thermodynamic equilibrium between the fluid and the porous matrix is assumed.
- The initial moisture content and temperature repartition in the wall is uniform.

Energy and mass conservation equations can be expressed by [6] and [4]:

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + \rho C_m (\varepsilon h_{lv} + \gamma) \frac{\partial m}{\partial t} \tag{1}$$

$$\rho C_m \frac{\partial m}{\partial t} = D_m \frac{\partial^2 m}{\partial x^2} + \delta D_m \frac{\partial^2 T}{\partial x^2} \tag{2}$$

where, T is the temperature, m moisture potential, t time, C_p and C_m are heat and moisture capacities of the medium, k and D_m are thermal and moisture diffusion coefficients, respectively, ρ is the dry body density, ε the ratio of vapor diffusion coefficient to coefficient of total moisture diffusion, γ the heat of absorption or desorption, δ the thermogradient coefficient and h_{lv} the heat of phase change.

The moisture potential concept, based on thermodynamic similarities between heat and mass transfer in hygroscopic materials, has been introduced by Luikov [2] and recently developed by Qin *et al.* [6]. It is defined as a linear function of the moisture content w. It is expressed by:

$$w = C_m m$$

The dimension of specific mass capacity C_m is kg kg$^{-1\circ}$M, where$^\circ$M denotes a mass transfer degree. This relation explains that, in thermodynamic equilibrium condition (i.e. in hygrothermal equilibrium), there is a definite distribution of moisture in a porous media. With an increase in the total mass of moisture, its content in the separate parts of a body increases. As expected, the moisture transfer takes place from the higher potential toward the lower one. This moisture transfer inside pores can be attributed to the mechanism of liquid transfer under consideration: evaporation of liquid takes place at one meniscus of the pore and vapor consideration at the opposite one. It stands to reason that the quantity of liquid evaporating from one meniscus must be equal to the quantity of vapor condensation on the opposite meniscus.

To facilitate the writing and to appear easily, the Laplace transformation of the following groups were introduced from Eq. (1) and (2):

$$L = \frac{k}{\rho C_p} \tag{3}$$

$$D = \frac{k D_m}{\rho C_m [k + D_m \delta (\varepsilon h_{lv} + \gamma)]} \tag{4}$$

$$\nu = \frac{C_m (\varepsilon h_{lv} + \gamma)}{C_p} \tag{5}$$

$$\lambda = \frac{C_p D_m \delta}{C_m [k + D_m \delta (\varepsilon h_{lv} + \gamma)]} \tag{6}$$

Thus the following simplified equations are obtained. These equations are similar to those obtained by Chang and Weng [9]:

$$L \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} - \nu \frac{\partial m}{\partial t} \tag{7}$$

$$D \frac{\partial^2 m}{\partial x^2} = \frac{\partial m}{\partial t} - \lambda \frac{\partial T}{\partial t} \tag{8}$$

Table 1
Experimental drying conditions and product properties.

Property	Value
Cm(K$_g$/Kg$^\circ$M)	0.01
C_p(J/(KgK)	2500
δ($^\circ$M/K)	2
k(W/mK)	0.65
ρ(K_g/m^3)	370
Dm(K$_g$/ms$^\circ$M)	2.2×10^{-8}
T_b($^\circ$C)	10
T_i($^\circ$C)	60
T_0($^\circ$C)	110
$l(m)$	0.024
hl_{tv}(J/Kg)	2.5×10^{-9}
m_b($^\circ$M)	86
m_i($^\circ$M)	45
m_0($^\circ$M)	4
γ	0
ε	0.3