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Multiple steady states in combined buoyancy and wind driven natural ventilation: The conditions for multiple solutions and the critical point for initial conditions

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Abstract

Natural ventilation systems may have multiple steady states in the combined buoyancy and wind driven mode in a single space. In this paper, dynamical system analysis is applied both graphically and quantitatively to investigate the multiple steady state behavior of such a natural ventilation system.

The necessary conditions for the system to have multiple steady states are derived for a single space combined wind and buoyancy driven natural ventilation system. Distinct from previous studies, the derivation comes from the transient dynamical system behaviors of the system rather than from the steady state equations, and it can be applied to more general cases.

Based on the dynamical system analysis, the quantitative relation between the initial temperature and the final steady state is investigated for the single space case. The unstable steady state, which cannot physically exist in reality, is the critical point of the initial value in determining the final (steady) state of the system.

Keywords: Multiple steady states; Dynamical system; Initial values; Necessary conditions

1. Introduction

Multiple steady state behavior occurs in many non-linear engineering dynamic systems (e.g., [1,2]). Such multiple steady state behavior exists in some natural ventilation systems for buildings due to the non-linearity of the natural ventilation system dynamics. Linden [3] investigated a naturally ventilated system with floor heating and stated the possibility of hysteresis on the system when the boundary condition changes gradually. Further experimental studies were conducted by Hunt and Linden [4] both theoretically and experimentally. Li and Delsante [5] investigated a natural ventilation system where wind opposes buoyancy effects as shown in Fig. 1. They solved the steady state equations that in some instances result in three possible mathematical solutions with the same boundary conditions. Similar results were also obtained by Gladstone and Woods [6] when investigating a naturally ventilated space with a heated floor. Among the three steady states, one is unstable and does not exist in reality while the other two are stable for small scale disturbances. Later, Li et al. [7] demonstrated several more cases in which multiple steady states can exist. A reduced scale experiments was also carried out on a single zone with blocking ventilation to prove the existence of two steady states. Heiselberg et al. [8] conducted both experimental and CFD simulations on the single space natural ventilation and proved the multiple steady states do occur in reality under certain conditions. Livermore and Woods [9] discovered the existence of three steady states in a two-variable buoyancy driven natural ventilation system, among which two are shown to be locally stable and the other is not.

An example of multiple steady state systems is the single zone natural ventilation system with opposing buoyancy and wind forces, shown in Fig. 1. Solving the steady state

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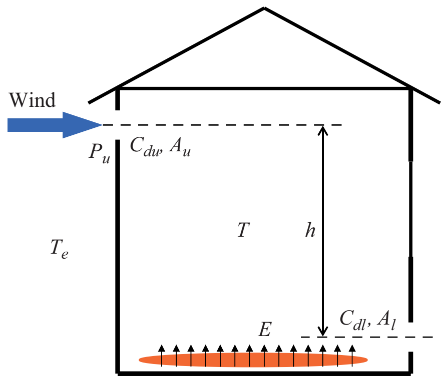

Fig. 1. Configuration of single zone combined buoyancy and wind driven natural ventilation.

equations may result in three different mathematical solutions under the same boundary conditions. Among the three steady state solutions, one is mathematically unstable while the other two are stable. The conditions for the multiple steady states to exist were also expressed for a special case where the wall is adiabatic ($\mathrm{UA} = 0$) to the outdoor air in a previous study [7]. The initial conditions of the nonlinear system were found important to the final state of the natural ventilation system when different steady states exist (e.g., [1,7,8]). For the system shown in Fig. 1, if the initial room temperature is relatively low, the final steady state of the system will be pressure dominated downward ventilation. If the initial room temperature is relatively high, the final state of the system may be buoyancy dominated upward ventilation.

However, in the previous studies, only steady state equations were used to derive the conditions for the existence of multiple steady states. These studies, therefore, did not systematically consider the transient dynamics of the system, which are critical to the system's behavior. In this paper, the transient dynamics of the system is investigated by a dynamical system method and their applications are demonstrated. For example, the dynamical system analysis can be used as a new method to study necessary conditions for the existence of multiple steady states, which was previously studied by solving steady state equation and can only easily applied to a special $\mathrm{UA} = 0$ case. This dynamical system method reflects the underlying governing dynamics and can be conveniently applied to the more general cases of $\mathrm{UA} \geqslant 0$.

More importantly, the dynamical system analysis quantitatively reveals the impact of the system's initial values on its final steady state. The "relatively low" or "relatively high" initial temperature argument about the initial values in the previous studies was a vague assertion, which did not demonstrate how exactly the initial temperature values can determine the final steady state. In this paper, we found the critical point of the initial temperature values that can determine the final steady state of the system. If the initial temperature is lower than this critical temperature, the system will go to one steady state (wind dominated downward); if it is higher than the critical temperature, the system will go to another steady state (buoyancy dominated upward). As will be shown in later sections, this critical temperature point is exactly the unstable steady state of the system, which was generally viewed as of no practical use in previous studies.

In summary, the dynamical system analysis in this paper will provide explanations on the existing conditions of multiple steady states and the dynamics mechanism of the initial value's impact on the final steady state of the system. The conclusions in this paper will provide guidelines for the explanations on the multiple steady states phenomena in airflow and thermal models such as multi-zone, zonal, and Computational Fluid Dynamics (CFD) models. Further more, the dynamical system analysis in this paper also builds the foundations for a further study on the state transitions behaviors between the multiple steady states under "strong" perturbations [10].

2. System equations

The basic configuration of a single zone with wind opposing buoyancy is shown in Fig. 1. Two openings are located at different heights and enable both wind-driven flows introduced by the external wind and buoyancy-driven flows introduced by the heat source. If the wind force is greater than the buoyancy force, the airflow will be wind dominated and downward. Otherwise, the airflow will be buoyancy dominated and upward. In this case we assume well-mixing air within the single space and power-law flow relations for the two openings. The thermal mass is assumed to be at the same temperature as the air if any is considered. Applying energy balance and empirical flow equations to the system yields:

Energy balance:

$$Mc_p \frac{\mathrm{d}T}{\mathrm{d}t} = q \cdot \rho_\mathrm{a} c_{p\mathrm{a}} \cdot (T_\mathrm{e} - T) + \mathrm{UA} \cdot (T_\mathrm{e} - T) + E. \quad (1)$$

Flow component:

$$q = \overline{C_\mathrm{d} A} \cdot \sqrt{\left| 2gh\beta \cdot (T - T_\mathrm{e}) - 2\frac{P_\mathrm{w}}{\rho_\mathrm{a}} \right|}. \quad (2)$$

The symbols are defined as follows:

M is the total thermal mass (including the air) in kg. t is the time in s. q is the volume flow rate of the air in m^3/s. ρ_a is the air density in $\mathrm{kg/m}^3$ (assume constant). c_p is the specific heat of the thermal mass (including the air) in J/kg K. $c_{p\mathrm{a}}$ is the specific heat of the air in J/kg K. T is the indoor temperature in K. T_e is the outdoor temperature in K. UA is the total conductance of the enclosure system in W/K. If the enclosure system is adiabatic, $\mathrm{UA} = 0$; otherwise, $\mathrm{UA} > 0$ in general cases h is the height difference between the upper and the lower openings in m. g is acceleration of gravity and is assumed to be constant as 9.8 $\mathrm{m/s}^2$. β is the thermal expansion coefficient of air in 1/K. For ideal gas, $\beta = 1/T$. P_w is the wind pressure difference