Case Studies in Structural Engineering 4 (2015) 1–13

Contents lists available at ScienceDirect

Case Studies in Structural Engineering

journal homepage: www.elsevier.com/locate/csse

Strain limits vs. reinforcement ratio limits – A collection of new and old formulas for the design of reinforced concrete sections

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ARTICLE INFO

Article history:
Available online 11 May 2015

Keywords:
Reinforced concrete
Unified design method
Reinforcement ratios
Compression controlled sections
Transition zone sections
Tension controlled sections

ABSTRACT

This paper presents a formulation for the design of reinforced concrete flexural members. The formulation yields exactly the same results as the current American Concrete Institute (ACI) design approach but it is based entirely on the concept of *reinforcement ratios.* This is in contrast to the current ACI approach which relies on strain limits [1]. A formulation based on reinforcement ratios is simpler and more intuitive and therefore has important pedagogical advantages. The formulation presented here can be thought of as an attempt to reconcile the new approach to design introduced by the ACI code in 2002, with the traditional approach to design that was in use from 1963 to 2002. The traditional approach to design of reinforced concrete sections uses the concept of reinforcement ratios. The new ACI approach, referred to here as the *unified design method* (UDM), requires consideration of rather cumbersome strain limits and/or geometric strain relationships. In this paper, it is shown that the UDM approach can be formulated much in the same way as the traditional approach, as long as a series of formulas involving reinforcement ratios are introduced. These formulas are presented in this paper. Many of them are well known, but some are new. In particular, a new formula for the *compression-controlled* reinforcement ratio limit, and a new direct procedure for the design of *transition-zone* sections are presented. The formulation presented in this paper should prove useful both for the instructor in the classroom, and for the practicing structural engineer. Derivation details for many of the formulas in the paper are given and several numerical examples to illustrate their use are provided at the end.

Introduction

In 2002, the ACI introduced a new approach to the design of structural members subject to bending. This change was introduced through the ACI-318-02 publication [2]. The motivation for the change was to achieve uniformity between the design procedures used for the design of pre-stressed, reinforced, and compression members [3]. As a consequence of this, the new approach to design has been termed unified design method (UDM) by some authors [4,5]. This name will be adopted in this paper. The UDM introduced the ideas of *tension-controlled, compression-controlled*, and, *transition-zone* beam sections. This characterization of reinforced concrete sections requires consideration of strain limits and/or geometric strain

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http://dx.doi.org/10.1016/j.csse.2015.05.001

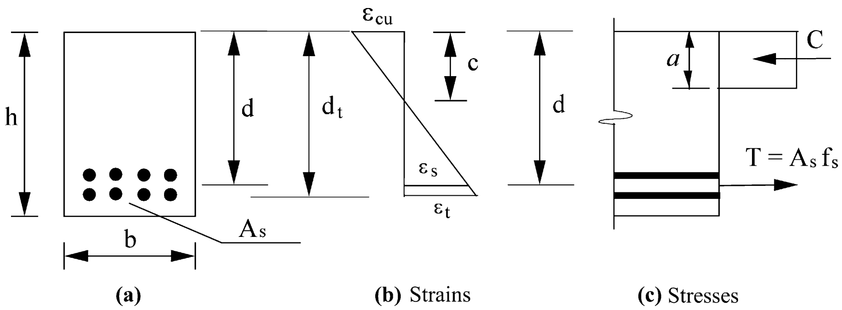

Fig. 1. Schematic representation of stresses and strains in a typical reinforced concrete section.

relationships. In the author's opinion, this characterization lacks intuitive and pedagogical appeal. The traditional approach to design of reinforced concrete sections relies instead on the concept of reinforcement ratios. The traditional approach can still be used (as of the ACI-318 2011 edition [1]) but it has been relegated to an appendix in the ACI code; and it is likely to disappear from it in subsequent editions [6]. The objective of this paper is to introduce a series of formulas for the design of flexural members that relies entirely on reinforcement ratios.

Tension-controlled, transition-zone, and compression-controlled sections

Fig. 1 shows the usual representation of the flexural stresses and strains in a typical reinforced concrete section. The notation and nomenclature are the usual, with c representing the depth of the neutral axis of the section; d representing the distance from the top of the section to the centroid of the main reinforcement; d_t the distance from the top of the beam to the bottom layer of the tensile reinforcement; a, the depth of the compression block; A_s, the area of the main reinforcement; and f_s, the stress in the tension steel.

Referring to Fig. 1b, we observe that:

$$\frac{c}{d_t} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_t} \tag{1}$$

On the other hand, from the stress diagram (Fig. 1c), we get the familiar expression for the depth of the compression block a, as:

$$a = \frac{A_s f_s}{0.85 f'_c b} \tag{2}$$

where f'_c represents the compressive strength of concrete, as usual.

Comparing now (1) and (2), recalling that $a = \beta_1 c$, and that the reinforcement ratio is defined as $\rho \equiv \frac{A_s}{bd}$, (see e.g., [7]) we conclude that:

$$\rho = \frac{0.85 \beta_1 f'_c}{f_s} \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_t} \right) \frac{d_t}{d} \tag{3}$$

Eq. (3) is a completely general relationship for singly reinforced concrete beam sections. This equation can be specialized to fit a number of important situations as it will be shown below.

Tension-controlled sections

The ACI-318-02 defines a tension-controlled section as a section such that the strain ε_t in the lowermost layer of steel is greater than or equal to 0.005. This is to ensure that the main steel yields well before the concrete crushes, providing enough ductility for the section even for seismic zones. This definition is inspired by the yield strain of grade 60 steel (G60 for short[1]) which is approximately equal to 0.002[2]. (The definition of tension-controlled section is the same for other grades of steel despite the fact that ε_y is different: for G75 steel for instance, $\varepsilon_y = 0.0026$). The European code (known as EC2) has similar provisions to guarantee adequate ductility of the sections (see e.g., [8,9]).

Now, by substituting $\varepsilon_{cu} = 0.003$ and $\varepsilon_t = 0.005$ in (1), one obtains:

$$\frac{c}{d_t} = 0.375 \tag{4}$$

[1] The grade is equal to the yield strength of the steel in the corresponding units. For instance, in metric units G420 steel has a yield strength of 420 MPa (i.e., $f_y = 420$ MPa). This steel is practically equivalent to G60 steel which has a nominal yield strength of 60 ksi (i.e., $f_y = 60$ ksi).

[2] Actually, $\varepsilon_y = 0.00207$ for G60 steel (assuming a modulus of elasticity $E_s = 29{,}000$ ksi).