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An extended force density method for form finding of constrained cable nets

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ABSTRACT

The force density method (FDM) is a classical method used in linear and nonlinear form. The linear approach presents a quick tool for finding cable net new shapes by solving a set of linear equilibrium equations for certain topology, boundary conditions and assumed cables force density. The nonlinear approach was introduced to solve cable nets under constraints (assigned certain distance between nodes, limit force or unstressed length in some elements). Any type of constraint introduces nonlinearity.

This paper studied the prestressed cable nets and the loaded cable nets. For prestressed cable nets, coordinate constraints to all nodes of the cable net are introduced to modify the shape after graphically examining the preliminary shape. This preliminary shape resulted from linear analysis of assumed distribution of cable force densities. For analyzing cable nets under different load cases, the first load case is analyzed to achieve the coordinate constraints assigned to nodes. Analysis results are node coordinates, cable forces and lengths. Young's modulus and areas of cables are used to calculate the unstressed length of all cables using materialization equations, those lengths are used as constraint in the analysis of other load cases. Forces in all cables under different load cases/combinations are calculated. By using this approach, design of cable net under static load is simplified.

Introduction

Cable nets are structures consisting of cables or bars connected by pins or hinges, where cable elements can carry axial forces only. They have several advantages over traditional construction such as, light weight, high stiffness, elastic behavior and innovative forms. They also allow natural light in case of roofs. Its form depends on its forces and vice versa. This means strong relationship between force and form. Accordingly it could not be dealt with as conventional structures. The cable net shapes, whether pretensioned or not, should be analyzed to get the initial form which is a basic form for the design. Before 1970's, the only possible way was to build a physical model and use photogrammetric tool to measure the deformation under external loads. Linkwitz and Scheck (1971) [8], and Scheck (1974) [5] introduced the force density method. It is a simple and very powerful tool to get the initial form for a given cable connectivity, fixed points and cable force densities by using a system of linear equations to get the exact form and improve it to get the desired shape. Frank Baron (1971) [4] introduced nonlinear analysis of cable structures with the finite element method using stiffness matrix including geometrical matrix. Pellegrino and Calladine (1986) [13], Pellegrino (1990) [14] and Pellegrino (1993) [15] used singular value decomposition

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in mixed structure analysis. Dynamic Relaxation method developed by Day (1965) [2] to analyze concrete pressure vessels, Rayleigh (1967) [11] used the DR concept to obtain static equilibrium at the steady state.

Since its introduction, the form finding using FDM has been extensively researched. Some of the research points are topological mapping method [3], equal distribution of static or quasi-static life loads in the composites retainers of a foot bridge [1], equating sum of coordinate difference/length = 0 to obtain minimal way net [9], tension structures with integrated, linear, actively bent elements [16] and form-finding of cable domes making full use of symmetry [17], mixed cable and strut [6], tensegrity [7], membrane structures using force density and stress density and Dome erection analysis [18].

The great advantage of the FDM is that it is the only method among form findings that does not need cable geometry in advance. Thus, it enables the creation of lots of structural shapes based on their topology by converting the nonlinear relation between force and form to linear relation between cable force densities (force/length) and free joints coordinates.

Computation model

Cable nets are geometrically nonlinear and elastically linear. They sustain large displacements with small deformation in elements. Force density analysis assumes that cables are weightless and their joints are pinned. The analysis may be classified into linear and nonlinear.

The linear analysis is applied when a certain topology, known coordinates of fixed nodes and force densities in elements are known. The stability equations of force densities in Cartesian Coordinates forming a set of linear equations. Those coordinates enable the display of cable net and enable for improvement of this shape by changing fixed point location or position and/or force densities. Distribution of force densities not their values governs the form.

The nonlinear analysis comes from any limitation or constraints applied to cable nets. The constraints are typically not applied to all elements or nodes. The number of nonlinear equations is usually less than the number of cables or nodes and there might be more than one solution. Among those solutions, the one that achieves the constraints with a given tolerance is chosen. The previous researches dealt with fixed distance between nodes, limited force and unstressed length of some cables [5] and reactions [10,12] with defined values in chosen fixed nodes constraints.

Also, minimum way net is a cable net in which forces in all cables are equal. In this analysis, after calculating the cable force using LFDM, all forces are restrained to a chosen value and nonlinear analysis is performed.

In this research, the coordinate constraints are used to enhance and control the form of the initially found form from the LFDM. The coordinate constraints are divided into three different functions in x, y and z or more depending on coordinate combinations. The NLFDM with least square method is used to analyze the constrained cable net. The coordinate constraint is used in both prestressed and loaded cable nets. For loaded cable net, the coordinate constraint is used to control the deformation under loads in first load case and, in the following load cases the unstressed length calculated from the first load case is used as a constraint for the other load cases/combinations.

Force density method

FDM utilizes a specified net topology, fixed point coordinates and force densities of elements to get the coordinates of the free joints at equilibrium state. In this research, plan graph at z equal zero is used to define the topology of the net. Only fixed joints coordinates should be exactly defined. The notations used in describing stability equations are consistent with those used by Scheck (1974) as follows:

Lower case is a vector whereas upper case is a matrix. The same vector symbol in upper case refers to a diagonal matrix having vector elements as its diagonal. Element joints are i and j.

Stability equation

Fig. 1 shows system axis and coordinates. In this system only three stability equations exist and sum of forces in the three Cartesian directions are zero.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0 \tag{1}$$

Stability equations at a free joint K are as follows:

$$S_a \cos(a,x) + S_b \cos(b,x) + S_c \cos(c,x) + S_d \cos(d,x) = P_x \tag{2a}$$

$$S_a \cos(a,y) + S_b \cos(b,y) + S_c \cos(c,y) + S_d \cos(d,y) = P_y \tag{2b}$$

$$S_a \cos(a,z) + S_b \cos(b,z) + S_c \cos(c,z) + S_d \cos(d,z) = P_z \tag{2c}$$

Where S_a is the force in element a, cos (a, x) is the cosine of the angle between element a and X axis, and P_x is the x component of the applied force at K. Similar equations are derived for Y and Z directions.

Substituting cosine by the normalized projection length, Equations 2 become: