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Highly anisotropic corrugated laminates deflection under uniform pressure

C. Thurnherr *, Y. Mirabito, G. Kress, P. Ermanni

Laboratory of Composite Materials and Adaptive Structures, Department of Mechanical and Process Engineering, ETH Zürich, Tannenstr. 3, CH-8092 Zürich, Switzerland

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ABSTRACT

Corrugated laminates with large corrugation amplitude have highly anisotropic stiffness properties. Here, the bending response of rectangular corrugated laminates to homogeneous transverse pressure is investigated. The center-point deflection of thin laminates with high corrugation amplitude is controlled by the higher bending-stiffness value alone. Simple formulas for calculating the center-point deflection are developed from beam formulas by replacing beam bending stiffness with plate bending stiffness and the line load with pressure. A finite-element model is used to verify the results and to illustrate the peculiar load-carrying action of the highly anisotropic plate. Further an error analysis is presented to indicate for which stiffness and aspect ratios the beam-equation modeling approach is valid and can be used for preliminary design.

1. Introduction

Morphing wing solutions are a promising approach to improve the flight performance of an aircraft using lower weight than conventional airplanes. To achieve an adaptive behavior of the wing, elements are needed that have a high compliance in chord direction while they behave stiff in span direction [1–3]. The anisotropy is needed to guarantee that a good compliance in chord direction is achieved with applying only low actuation forces. On the other hand the structure still needs to sustain structural and aerodynamic loads which requires high bending stiffness about the chord direction. Corrugated structures are ideal candidates to fulfill these requirements [4–6]. They provide the required stiffness behavior and they can achieve high global axial strains since the local strains still remain relatively small thanks to the curved structure [7]. Flexible skins need to be designed in a way that they can withstand the external pressure during flight and too high out-of-plane deflections must be prevented [8,9]. Beside the anisotropy between the compliance in and the bending stiffness about chord direction, there is also an anisotropy between the bending stiffness about span and chord direction especially if the corrugation pattern have high amplitudes. This anisotropy also must be analyzed and considered to achieve optimal designs. To the authors' knowledge this has not been addressed in other studies so far. This also rises the question where and how the corrugated panels must be supported in order to keep the deflection due to aerodynamic pressure in a feasible range.

Finite element analyses of corrugated plates made from anisotropic material as suggested by other authors [10] can be very costly if the structure contains many periods and consists of many layers. Various models exist to analyze the equivalent stiffness of a corrugated laminate [11,12,6,13,14]. In the present paper we combine the model suggested by Kress and Winkler [13] with a plate model for rectangular anisotropic plates under uniform pressure [15] to estimate the maximum deflection due to aerodynamic pressure which is crucial for the design of corrugated laminates for morphing wings. However, a closed-form solution of the plate model could only be found for the simply supported load case. For clamped edges on the other hand, a new approach is needed that allows a fast and easy use for preliminary design.

The objective of the present work is to find methods that allow preliminary design of corrugated laminates under uniform pressure for morphing wings. We seek for procedures that are easy to apply and hence allow a fast first design estimation. In the following section we present the design problem, then an approach for simply supported plates is presented. Then we show how the solution for plates converges to beam theory for highly anisotropic plates. In the last section the suggested modeling approach is validated using commercial finite element method.

* Corresponding author.
E-mail address: thclaudi@ethz.ch (C. Thurnherr).

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2. Theory and investigation

2.1. Corrugated laminate as flexible skin

The geometric effects of corrugation induce highly anisotropic stiffness properties to corrugated sheets. For instance, the membrane stiffness along the corrugations is much smaller than that along the direction transverse to them. The corrugated sheet is thus a candidate for a flexible skin as part of a morphing-wing design for aircraft if the corrugations are along the wing chord direction. Then, the airfoil shape can change thanks to the high compliance along the chord direction whilst the same corrugated sheet makes a significant contribution to the required structural stiffness along the span direction. Fig. 1 indicates the principle of including a flexible skin into an airplane-wing design, defines the span and the chord directions, and indicates the orientation of the corrugated sheet. Even though the corrugation geometry induces much higher degrees of anisotropy than fiber-reinforced composite materials possess, the use of the latter for a base sheet material is recommended as the limit strain of composites is significantly higher than that of metals. The higher limit strain allows for larger changes of length along the corrugations.

2.2. Deflection due to aerodynamic pressure

The aspect which seems to not yet have been addressed in the literature is the deflection, or out-of-plane displacement, of the flexible skin due to aerodynamic pressure. For simplicity the flexible skin is considered with a rectangular shape with side lengths a and b and the aerodynamic pressure p_0 is assumed to be homogeneous. A measure of the skin deformation is the maximum deflection w_{max}. As indicated in Fig. 1, it occurs at its center $x = a/2$ and $y = b/2$, where x and y are the coordinates along the span and chord directions, respectively. The deflection depends on the pressure, the skin size, the boundary conditions, and the bending stiffness values. The bending stiffness values in turn depend on the sheet material and thickness and on the corrugation shape. The corrugation shifts material fibers away from the neutral plane which is also the basic principle of sandwich constructions [16]. It is therefore anticipated that shear deformation may contribute significantly to deflection.

2.3. Corrugated laminate deflection analysis

The combination of simulation tools used for calculating the deflections of rectangular plates made of corrugated laminates is depicted in Fig. 2. The exact solution to the problem of calculating the bending deflection w_b of rectangular anisotropic plate, simply supported at all four edges, under homogeneous pressure is found in the textbook by Jones [15]:

$$w_b = \frac{16p_0}{\pi^6}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{mn}\cdot\frac{\sin(m\pi\frac{x}{a})\sin(n\pi\frac{y}{b})}{D_{11}\left(\frac{m}{a}\right)^4+2(D_{12}+2D_{66})\left(\frac{mn}{ab}\right)^2+D_{22}\left(\frac{n}{b}\right)^4}, \quad (1)$$

where D_{11} and D_{22} are bending stiffness values with respect to the x and y directions of the plate, D_{12} reflects the stiffening effect of the constrained Poisson-ratio effect, and D_{66} is torsional stiffness. It holds for laminates without coupling between bending and twist, or $D_{16} = D_{26} = 0$, so that variables can be separated into functions of plate coordinate x alone times functions of y alone [17]. The textbook by Zenkert [16] provides, in addition, an exact solution for the transverse-shear deflection w_s:

$$w_s = \frac{16p_0}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{mn}\,\frac{\sin(m\pi\frac{x}{a})\sin(n\pi\frac{y}{b})}{K_{44}\left(\frac{m}{a}\right)^2+K_{55}\left(\frac{n}{b}\right)^2}, \quad (2)$$

where K_{44} and K_{55} are transverse shear stiffness values with respect to the x and the y axes of the substitute homogeneous plate. For deflections being much smaller than plate in-plane dimensions, the concept of partial deflections known from sandwich theory [16] may be used:

$$w = w_b + w_s. \quad (3)$$

The formulas (1) and (2) require the data input indicated at the right of Fig. 2. The bending and the transverse-shear stiffness values of the corrugated sheet are marked with a tilde. They characterize the structural load response of a corrugated plate, which on a large scale behaves like a homogeneous plate having the same stiffness values. The substitute-plate stiffness values of a corrugated laminate are calculated with the formulas given by Kress and Winkler [13], for which a FORTRAN program CORLAM (Corrugated Laminates) exists at our Lab *CMAS*. This model considers corrugation shapes consisting of circular sections as Fig. 3 indicates. The shape shown in the figure corresponds with the maximum amplitude c for a given unit-cell length P, see Fig. 4 for definition of c and P. The amplitude is constrained by the fact that the laminate may not penetrate itself; it gives the highest degree of anisotropy and, at the same time, smoother aerodynamic surfaces than lower amplitudes, for instance the amplitude corresponding to semi-circles. The data required by the model include unit-cell length and said corrugation amplitude and the stiffness properties of the base laminate.

The stiffness properties of the base laminate are contained in the ABD matrix. The ABD matrix connects mid-plane strains ϵ^0 and bending curvature κ with line forces $\mathbf{N}$ and line moments $\mathbf{M}$. It consists of the membrane stiffness matrix $\mathbf{A}$, the bending stiffness matrix $\mathbf{D}$, and the coupling stiffness matrix $\mathbf{B}$. In the case of a symmetric laminate the coupling stiffness matrix $\mathbf{B}$ vanishes, and for a cross-ply laminate coupling exists neither between extension and in-plane shear nor between bending and twist, which is reflected by the sub-matrices $\mathbf{A}$ and $\mathbf{D}$ having the following population structure:

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}. \quad (4)$$

The ABD matrix of the base laminate is calculated with the classical theory of laminated plates which is also found in the textbook by Jones [15]. A proprietary FORTRAN program *CAP* (Composites Analysis Program) for evaluating the classical theory of laminates plates is available at our lab *CMAS*.

The transverse-shear stiffness values K_{44} and K_{55} connect plate shear angles γ_{xz} and γ_{yz} with transverse line forces T_x and T_y, respectively. As shear stress distributions must satisfy zero-traction natural boundary conditions at the laminate surfaces and are stack-up sensitive, the calculation of K_{44} and K_{55} according to first-order shear theory can be quite inaccurate and a more refined theory has been provided by Rolfes and Rohwer [18]. We do not yet have developed a method for exact calculation of the corrugated laminate homogenized substitute plate transverse-shear stiffness values $\tilde{K}_{44}$ and $\tilde{K}_{55}$. Our estimate bases on the rough model illustrated with Fig. 4, which combines the contributions of

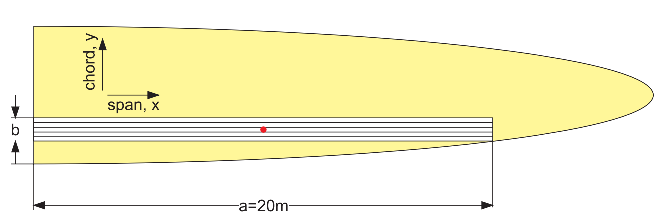

Fig. 1. Wing with flexible skin.