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A serendipity plate element free of modeling deficiencies for the analysis of laminated composites

João Elias Abdalla Filho [a,*], Ivan Moura Belo [b], John O. Dow [a]

[a] Department of Civil, Environmental and Architectural Engineering, University of Colorado, Boulder, CO, USA
[b] Mechanical Engineering Program, Federal Technological University of Paraná (UTFPR), Curitiba, Paraná, Brazil

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ABSTRACT

An eight-node serendipity element free of shear locking and spurious zero-energy modes is formulated to model laminated composite plate problems. The element is based on a first-order shear deformation theory and on the equivalent lamina assumption. Stresses are calculated through the thickness of the plate. As the model is only capable of representing transverse shear strains and stresses as constants, while their actual variations are parabolic, a shear correction factor is used. The element is formulated using strain gradient notation, which is a physically interpretable notation that allows for a detailed *a-priori* analysis of the finite element model. The element's shear strain polynomials are inspected, and the spurious terms which are responsible for shear locking are identified. The element is corrected by simply removing such spurious terms from those shear strain expansions. Further, the compatibility modes are also clearly identified and maintained in the shear strain expansions in order to prevent the introduction of spurious zero-energy modes. Numerical results show the shear locking effects caused by the spurious terms on displacement and transverse stress solutions. They also show that properly refined meshes composed of corrected elements provide solutions which converge rather well for moderately thick to thin plates.

1. Introduction

Research on modeling and analysis of laminated composite structures has been very active in at least the last four decades. The concern to accurately represent the actual behavior of this kind of structure has led researchers to develop analytical and numerical models every time more refined. Thorough reviews on theories and finite element models for laminated composites have been presented in the literature [1–4]. Carrera [5] discusses theories for laminated composites in the perspective of the correct representation of the continuity of the transverse displacement and the equilibrium of interlaminar transverse stress components. The author indicates that layerwise models are necessary for an accurate evaluation of the normal transverse stresses. Another work presents a review on the different methods, both analytical and numerical, available for estimating transverse and interlaminar stresses in laminated composite plates and shells [6]. An early attempt to formulate a finite element for laminated composites made by Mawenya and Davies [7] resulted is an element based on first-order shear deformation theory (FSDT) with rotations varying from layer to layer. Another early example is the work by Panda and Natarajan [8] where it is presented an element based on the eight-node degenerate shell developed by Ahmad et al. [9]. As the first-order shear deformation theory is viewed as a limiting theory in representing accurately the behavior of laminated composites, several other works have proposed high-order deformation theories (HSDT) [10–14]. An overview of the relationships between classical and shear deformation theories has been presented [15]. Computational models ranging from simple to refined have been developed to perform numerical evaluation of all those theories [14,16]. The concern with the development of accurate laminated composite finite element models obviously has included avoiding shear locking and other spurious mechanisms. Isoparametric elements require some form of reduced-order integration of the stiffness matrix. Mawenya and Davies [7], for instance, followed the recommendation of using a 2×2 Gaussian integration provided by Zienkiewicz et al. [17]. It was not known at the time that such a strategy would not be very efficient as it would be proved later by other researchers. The works of Hughes and

* Corresponding author at: Mechanical Engineering Program, Pontifical Catholic University of Paraná, Brazil, and Civil Engineering Program, Federal Technological University of Paraná, Brazil.
E-mail address: joao.abdalla@pucpr.br (J.E. Abdalla Filho).

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co-workers with different plate elements shed light over the shear locking problem as well as spurious zero-energy modes (rank deficiency) and made significant progress in developing selective reduced-integration procedures for isoparametric elements [18–20]. However, as selective reduced integration of Lagrange elements introduces spurious zero-energy modes, the Heterosis element was devised to circumvent such problem at the expense of a more complex formulation [21]. Alternative formulation procedures have been devised seeking the development of better plate and shell elements. Among those we cite the field consistency approach [22] and the mixed interpolation of tensorial components [23]. Laminated composite shell elements have been developed using the latter [24]. Also, an approach of adding a shear correction factor to transverse shear strain expressions developed by Verwoerd and Kok [25] has been applied to three-node triangular laminated composite plate elements based on FSDT to suppress shear locking effects [26]. Zhang and Yang [27] introduce triangular and quadrilateral plate and shell elements for linear and nonlinear analysis of thin to moderately thick laminates. Such elements are shear locking-free due to the use of Timoshenko's beam functions to represent deflections and rotations of the elements sides. A C^1 six-node triangle is developed using trigonometric functions to represent transverse shear stresses. Polynomial orders for the transverse displacement and for the rotations are selected such that transverse shear strain compatibility is attained, thus avoiding shear locking naturally [28]. Kim et al. [29] formulate a FSDT shell element within the co-rotational framework for geometrically nonlinear analysis. The ANS (Assumed Natural Strains) method is employed, precluding shear locking effects, and thus allowing good behavior for thin and thick problems. Further advances include layerwise models [30], and elements based on Reddýs simple higher-order shear deformation theory [31,32]. Moleiro et al. [33] develop a layerwise finite element model based on a mixed least-squares formulation which enables interlaminar continuity of displacements and transverse stresses in the form of C^o continuous functions. Mantari and Guedes Soares [34] introduce a generalized layerwise displacement-based HSDT and the corresponding finite element in which the number of DOFs is limited for being independent of the number of layers. Pandey and Pradyumna [35] present a C° layerwise eight-node finite element model for three-layered composite plates where first-order displacement fields are assumed for the top and bottom layers while a higher-order displacement field is assumed for the middle layer. Also, there are contributions made using the Refined Zigzag Theory (RZT), which has been presented by Tessler et al. [36] recently. RZT is based on FSDT and employ piecewise-linear zigzag functions that provide better representations of the deformations states of transverse shear-flexible plates. Those authors [37] present two- and a three-node beam elements that avoid shear locking effects by employing anisoparametric interpolations to approximate the kinematic variables that are necessary to model planar deformations. The same group [38], following the same line of the previous referenced work, formulate RZT-based six- and three-node triangular plate elements, which also employ shape functions based on anisoparametric interpolations. Further, Eijo et al. [39] present the formulation of an isoparametric four-node C° quadrilateral plate element based on the RZT theory. Shear locking is avoided by using an assumed linear strain shear field. Finally, another approach is the one adopted by Natarajan et al. [40] which combines Carrera's Unified Formulation (CUF) and the cell-based smoothed finite element method. A four-node quadrilateral based on the field consistency requirement is employed to eliminate shear locking.

This paper describes the development of a FSDT-based eight-node serendipity plate finite element using strain gradient notation and makes an assessment of its capabilities in the analysis of laminated composite plate problems. Spurious terms which are the cause of shear locking are identified and removed.

2. On the strain gradient notation

Strain gradient notation (Dow [41]) is a physically interpretable notation which relates displacements to the kinematics quantities of the continuum. It has been used as an alternative procedure for formulating finite elements and also finite difference templates. The relationships between displacements and the kinematics quantities are derived through a procedure which identifies the physical contents of the polynomial coefficients. An important feature of strain gradient notation is that the modeling characteristics of the finite element become apparent to the developer since the early steps of the formulation. This allows for the sources of modeling errors to be identified and consequently removed from the finite element shear strain polynomial expansions prior to the formation of the element stiffness matrix. In this paper, sources of shear locking (parasitic shear terms) are precisely identified and removed from the shear strain polynomial expansions of the eight-node serendipity plate element. Causes of spurious zero energy modes are explained, and this deficiency is avoided by correctly eliminating shear locking. Parasitic or spurious shear terms which are present in the shear strain polynomial expansions of the serendipity plate element are identified by inspection. It is demonstrated both theoretically that they are flexural terms which cause locking of the model by increasing the elements shear strain energy unduly when the plate undergoes bending. Such terms are then simply removed from the shear strain polynomials, avoiding shear locking to occur. It is also demonstrated that spurious zero-energy modes are not introduced into the model by recognizing and not removing the compatibility modes. Such modes can be easily confused with shear locking terms and, therefore, be inadvertently removed. This is the limitation of reduced-order integration schemes in attempting to correct elements for locking. Along with eliminating legitimate spurious terms which are responsible for locking, those techniques also eliminate compatibility modes, thus introducing spurious zero-energy modes. Hence, the use of strain gradient notation has the advantages that locking is taken care of correctly and *a-priori* of the formation of the stiffness matrix and of the computer implementation, and that the formulated element is of correct rank since no spurious zero-energy modes are introduced. Formulation associated to strain gradient notation is described in the next two sections.

3. First-order shear deformation theory

The kinematic relations according to the Reissner-Mindlin or first-order plate theory may be written as follows:

$$u(x,y,z) = u_o(x,y) + z\theta_x(x,y) \tag{1}$$

$$v(x,y,z) = v_o(x,y) - z\theta_y(x,y) \tag{2}$$

$$w(x,y) = w \tag{3}$$

$$\theta_x(x,y) = \frac{\partial u(x,y,z)}{\partial z} \tag{4}$$

$$\theta_y(x,y) = -\frac{\partial v(x,y,z)}{\partial z} \tag{5}$$

where u and v are in-plane displacements along the x and y directions, respectively, w is the out-of-plane (normal to the middle surface) displacement, θ_x and θ_y are rotations in the x and y directions, respectively (or around the y and x axes, respectively), and u_0 and v_0 are the middle surfacés in-plane displacements along the x and y