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MITC9 shell finite elements with miscellaneous through-the-thickness functions for the analysis of laminated structures

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ABSTRACT

This paper focuses on developing and exploiting the potential of miscellaneous through-the-thickness approximating functions for FEM analysis of laminated composite plates/shells. Considering the theory of series expansion, Taylor series, trigonometric series, exponential functions, and miscellaneous expansions are implemented in the equivalent single layer models of Carrera Unified Formulation (CUF). Their performances in obtaining a good approximation of stress distribution through the thickness of the plate/shell are investigated by performing several static mechanical studies, and the inclusion of Murakami's zig-zag function is also evaluated. The results are compared with layer-wise theories in the framework of CUF by adopting as thickness functions both Legendre polynomials and Lagrange interpolations on Chebyshev nodes (Sampling-Surfaces method, SaS). The governing equations are derived from Principle of Virtual Displacement (PVD) and Finite Element Method (FEM) is adopted to get the numerical solutions. Nine-node 2D elements for plates and shells are employed, using Mixed Interpolation of Tonsorial Components (MITC) method to contrast the membrane and shear locking phenomenon. Simply-supported cross-ply plate and shell structures with various lay-ups and span-to-thickness ratios subjected to transverse bi-sinusoidal pressure load are analyzed. The results show that all the refined kinematic theories are able to capture the exact solution if a sufficient number of expansion (number of terms in the expansion of the displacement field) is taken, but the maximum computational cost can change for the different types of models. In some cases, combinations of different expansion theories (miscellaneous expansions) can show a significant reduction of computational costs.

1. Introduction

Thin-walled structures like composite plates and shells play an important role in aerospace engineering. Effective mechanical models should be able to capture the elastic behaviors of multilayered structures. Except for anisotropy, transverse shear stress calculation is an important issue for layer interfaces in laminated structures. Finite Element Method (FEM) has been widely used in engineering practice to obtain numerical approximation instead of mostly un-achievable exact analytical solutions. Given the great achievements of FEM over the past several decades in engineering application, innovative analysis methods and refined FEM models with better performances and less computational costs have been always needed.

According to published research, various theories for composite structures have been developed. They can be classified as: Equivalent Single Layer (ESL), in which the number of unknowns is independent of the number of layers, and Layer-wise approach (LW), in which the number of unknowns is dependent on the number of layers. The majority of early FEM calculations were performed with the classical Kirchhoff–Love theory and some examples are given in [1–5]. But, it was difficult to satisfy the requirements of compatibility in thin shell analysis because the rotations were derived from the transversal displacement. For this reason, plate/shell elements based on the First-order Shear Deformation Theory (FSDT) were developed by Pryor and Barker [6], Noor [7], Hughes [8], Panda and Natarayan [9], Parisch [10], Ferreira [11] and many others.

Also a large variety of plate/shell finite element implementations of higher-order theories (HOT) have been proposed in the last twenty years literature. HOT-based C^0 finite elements (C^0 means that the continuity is required only for the unknown variables and not for their derivatives) were discussed by Kant and co-authors

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[12,13]. In [14–18], Polit et al. proposed a C^1 six-nodes triangular finite element in which the transverse shear strains are represented by cosine functions. This element is able to ensure both the continuity conditions for displacements and transverse shear stresses at the interfaces between layers of laminated structures. A comprehensive discussion of HOT-type theories and related finite element suitability has been provided by Tessler [19]. Many other papers are available in which HOTs have been implemented for plates and shells, details can be found in the books by Reddy [20] and Palazotto and Dennis [21].

Dozens of finite elements have been proposed based on zig-zag theories [22,23]. An application of Reissner Mixed Variational Theorem (RMVT) [24] to develop standard finite elements was proposed by Rao and Meyer-Piening [25]. A generalization of RMVT as a tool to develop approximate solutions was given by Carrera [26]. The obtained finite elements represent the FE implementation of the Murakami theory [27] and were denoted by the acronym RMZC, (Reissner Mindlin Zigzag interlaminar Continuity). Full extensions of RMZC to shell geometries have been done by Brank and Carrera [28].

Concerning trigonometric polynomial expansions, some plate and beam theories have been developed. Shimpi and Ghugal [29] used trigonometric terms in the displacements field for the analysis of two layers composite beams. An ESL model was developed by Arya et al. [30] using a sine term to represent the non-linear displacement field across the thickness in symmetrically laminated beams. An extension of [30] to composite plates was presented by Ferreira et al. [31]. A trigonometric shear deformation theory is used to model symmetric composite plates discretized by a meshless method based on global multiquadric radial basis functions. A version of this theory, with a layer-wise approach, was proposed by the same authors in [32]. Vidal and Polit [33] developed a new three-noded beam finite element for the analysis of laminated beams, based on a sine distribution with layer refinement. Recently, the same authors have dealt with the influence of the Murakami's zig-zag function in the sine model for static and vibration analysis of laminated beams [34]. Static and free vibration analysis of laminated shells were performed by radial basis functions collocation, according to a sinusoidal shear deformation theory in Ferreira et al. [35]. It accounts for through-the-thickness deformation, by considering a sinusoidal evolution of all displacements along the thickness coordinate.

Concerning exponential expansions, Mantari et al. [36] presented the static response of advanced composite plates by using a new trigonometric-exponential higher order shear deformation theory (HSDT). In previous works [37–39], they demonstrated that the inclusion of exponential function in the shear strain function produces results with good accuracy. Based on this experience, they combined the exponential function with the tangential shear strain shape to obtain an improved HSDT. Karama et al. [40] developed a refined theory containing the exponential functions in thickness coordinate in the displacement field that was called as exponential shear deformation theory. In [40,41], they used this new shear stress function in the form of the exponential function to predict the mechanical behaviour of multi-layered laminated composite beams. Aydogdu [42] developed a new exponential higher order shear deformation theory for the buckling analysis of cross-ply laminated composite beams and compared the results with the theory of Karama et al. [40].

In [43,44] Carrera et al. presented the static and free vibration analysis of laminated beams using various refined beam theories by expanding the unknown displacement variables over the beam section axes using Taylor type expansions, trigonometric series, exponential, hyperbolic and zig-zag functions. A companion work [45] dealt with the analysis of laminated composite and sandwich plates by considering similar miscellaneous polynomials along the thickness direction in ESL approach. In these works, finite elements based on Carrera Unified Formulation (CUF) were used [46,47]. In the framework of CUF, a large number of refined-theories can be implemented in both ESL and LW approach for the analysis of multi-layered structures. In particular, for the analysis of the plates, MITC9 finite elements were employed [48] in which the Mixed Interpolation of Tensorial Components (MITC) method [49–52] is adopted to contrast the membrane and shear locking phenomenon. The present paper represents the extension of these elements with miscellaneous kinematics to the analysis of shell structures.

Finite element implementations of layer-wise theories were also proposed by many authors, among which Noor and Burton [53], Reddy [54], Mawenya and Davies [55], Pinsky and Kim [56], Chaudhuri and Seide [57], Rammerstorfer et al. [58]. In previous works, the authors already implemented MITC9 finite elements based on LW approach for the analysis of laminates structures and they used Legendre polynomials as thickness functions in the CUF framework [59]. In the present work, they include among the LW theories of CUF shell elements the Sampling Surface (SaS) method, introduced by Kulikov and Plotnikova [60,61] for the accurate analysis of laminated composite structures and implemented in [62] for an hybrid-mixed four-node quadrilateral laminated composite plate element (note that the origins of the SaS method can be traced back to contributions of Kulikov [63] and Kulikov and Carrera [64] in which three, four and five equally spaced SaS are employed). In this case, Lagrange polynomials are employed as thickness functions and Chebyshev polynomial nodes together with top and bottom positions of the layer are used as interpolating points along the thickness of the plate/shell.

This paper analyzes cross-ply plates and shells with simply-supported edges and subjected to transverse bi-sinusoidal pressure loads. The governing equations in weak form for the linear static analysis of composite structures are derived from the Principle of Virtual Displacement (PVD). The results, obtained with the different models, are compared with 3D exact solutions provided in literature.

The manuscript is organized as follows: an overview of ESL, ZZ and LW theories developed within the CUF framework is given in Section 2; the MITC9 shell finite element is presented in Section 1 and the constitutive relations for laminated composite structures are provided in Sections 4; in Section 5, the governing equations in weak form for the linear static analysis of composite shells are derived from the PVD. In Section 6, the results obtained using the proposed CUF theories are discussed. Section 7 is devoted to the conclusions drawn about this work.

2. Carrera Unified Formulation

The main feature of the Unified Formulation by Carrera [26] (CUF) is the unified manner in which the displacement variables are handled. In the framework of the CUF, the displacement field is written by means of approximating functions in the thickness direction as follows:

$$\delta \boldsymbol{u}^k(\alpha,\beta,z) = F_\tau(z)\delta \boldsymbol{u}^k_\tau(\alpha,\beta); \quad \boldsymbol{u}^k(\alpha,\beta,z) = F_s(z)\boldsymbol{u}^k_s(\alpha,\beta)$$
$$\tau, s = 0,1,\ldots,N \tag{1}$$

where (α,β,z) is a curvilinear reference system, in which α and β are orthogonal and the curvature radii R_α and R_β are constant in each point of the domain Ω (see Fig. 1). The displacement vector $\boldsymbol{u} = \{u, v, w\}$ has its components expressed in this system. $\delta \boldsymbol{u}$ indicates the virtual displacement associated to the virtual work and k identifies the layer. F_τ and F_s are the so-called thickness functions depending only on z. $\boldsymbol{u}_s$ are the unknown variables depending on the coordinates α and β. τ and s are sum indexes and N is the order