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Dynamic response of functionally graded material shells with a discrete double directors shell element

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ABSTRACT

The purpose of the paper is to compute the dynamic behavior of functionally graded material (FGM) shell structures subjected to time-varying excitation using 3D-shell model based on a discrete double directors shell element. The third-order shear deformation theory is introduced in the present method to remove the shear correction factor and improve the accuracy of transverse shear stresses. Material properties of the shell are assumed to be graded in the thickness direction by varying the volume fraction of the ceramic and the metallic constituents using general four-parameter power-law distribution. The transient excitation is defined in the time domain and known at each time. The damping material is neglected and the time derivative is approximated by Newmark method. Numerical results for deflection and stresses are presented for plates and spherical caps. The effect of an imposed force on the response of the FGM shell is discussed. The numerical examples prove a good accuracy and reliability compared to the few results available in literature.

1. Introduction

The improved properties of composite materials have led to their use in space and aerospace applications. However, the abrupt change of the properties across the interface between different materials in conventional composite materials causes high interlaminar stresses leading to delamination. In addition, large plastic deformation at the interface may activate the initiation and the propagation of cracks in the conventional composite material. Functionally graded materials (FGMs), which are special kind of composite with a gradual transition of material properties from one material to another, are made to overcome the problems associated to the discontinuity in conventional composite. The most known FGMs are made of a transition alloys from metal at one surface to ceramic at the opposite surface. This kind of FGMs was introduced as ultra high temperature resistant materials for nuclear reactor, chemical plants, heat engine components and aerospace vehicles. The thermal resistance of FGM is due to low thermal conductivity of ceramic [1,2].

The static behavior of FGM plates was studied in literature with analytical and numerical methods. Thin FGM plate can be studied by a classical plate theory [3,4]. For thick FG plates, some models take into account the transverse shear effect by using the First-order Shear Deformation Theory (FSDT) [5,6]. In the FSDT theories, transverse shear is assumed to be constant through the shell thickness and thus require the computation of shear correction coefficients, [7–9]. In fact, shear correction coefficients are problem dependent and cumbersome. This limitation of the FSDT forced the development of High-Order Shear Deformation theory (HSDT) which includes the consideration of realistic parabolic variation of transverse shear stress through the shell thickness in the isotropic cases. There is no need of a shear correction factor when using a HSDT but equations of motion are more complicated to obtain than those of the FSDT. The HSDT are introduced to develop a plate or shell elements, [10–14] among authors. An other possibility to introduce a HSDT is by using the enhanced assumed strain formulation (EAS) in solid-shell elements with quadratic transverse shear enhancement, [15,16]. Recently, a unified formulation developed by Carrera and his co-workers (CUF), which can generate any refined theory, is developed in static and free vibration for laminate composites and FGM shells [17–21].

FGM shell structures are subjected to severe dynamic loads. Therefore, an assessment of natural frequency and transient response of structures seems required. Based on the Mindlin's first-order shear deformation theory, free vibration of functionally graded plates and cylinder was investigated by Hosseini-Hashemi et al. [22]. Using a discrete double directors shell element, free vibration analysis of FGM shell structures was studied by Wali et al. [12]. Based on layer-wise finite element, the dynamic

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response of functionally graded cylindrical shell was studied by Yas et al. [23]. Using a refined 8-node shell element, a forced vibration of FGM to arbitrary loading is investigated in [24]. The transient response of FGM plate is analyzed by [5].

The objective of this work is to present a general formulation for forced vibration of FGM shell using discrete double directors shell model. The formulation is investigated for the transient response of FGM shells. Square plate and spherical cap are used to compare the accuracy of the formulation. Transient deflection and axial stress are analyzed along the thickness.

2. Double directors shell model

In this section, the geometry and kinematic of double directors shell model are described. The reference surface of the shell is assumed to be smooth, continuous and differentiable. Variables associated with the undeformed state will be denoted by upper-case letters and by a lower-case letter when referred to the deformed configuration.

2.1. Double directors shell kinematic assumption

Parameterizations, which define material points of the shell, are carried out in terms of curvilinear coordinates $\boldsymbol{\xi} = (\xi^1, \xi^2, \xi^3 = z)$. The position vectors of any material point (q), whose normal projection on mid-surface is the material point (p), in the initial states C_0 are given by:

$$\boldsymbol{X}_q(\xi^1, \xi^2, z) = \boldsymbol{X}_p(\xi^1, \xi^2) + z\boldsymbol{D}(\xi^1, \xi^2), \quad z \in [-h/2, h/2], \tag{1}$$

where h is the thickness. $\boldsymbol{X}_p$ and $\boldsymbol{D}$ are respectively a point of the reference surface and the initial shell director. The covariant basis $(\boldsymbol{G}_1, \boldsymbol{G}_2, \boldsymbol{G}_3)$ is obtained from the position vector by $(\boldsymbol{G}_1, \boldsymbol{G}_2, \boldsymbol{G}_3)$= $(\partial \boldsymbol{X}_q/\partial \xi^1, \partial \boldsymbol{X}_q/\partial \xi^2, \partial \boldsymbol{X}_q/\partial z)$, which yields in base vectors relative to the initial state:

$$\boldsymbol{G}_\alpha = \boldsymbol{A}_\alpha + z\boldsymbol{D}_{,\alpha}; \quad \boldsymbol{G}_3 = \boldsymbol{D}, \quad \alpha = 1, 2. \tag{2}$$

The surface element dA in the initial state is given by:

$$dA = \sqrt{A} dA_\xi, \quad \sqrt{A} = \|\boldsymbol{A}_1 \wedge \boldsymbol{A}_2\|, \quad dA_\xi = d\xi^1 d\xi^2. \tag{3}$$

The covariant reference metric tensor $\boldsymbol{G}$ at a material point ξ is defined by:

$$\boldsymbol{G} = \left[\boldsymbol{G}_i \cdot \boldsymbol{G}_j\right], \quad i, j = 1, 2, 3. \tag{4}$$

For later use, the geometrical variable $Det(\boldsymbol{G})$ and dV are related by $dV = \sqrt{(G)} d\xi^1 d\xi^2 dz$, where $Det(\boldsymbol{G}) = \sqrt{(\boldsymbol{G})} = \sqrt{(|\boldsymbol{G_{ij}}|)}$.

With the assumption of a double directors shell model, the position vector of any point q, in the deformed configuration is given by (show Fig. 1):

$$\boldsymbol{x}_q(\xi^1, \xi^2, z) = \boldsymbol{x}_p(\xi^1, \xi^2) + f_1(z)\boldsymbol{d}_1(\xi^1, \xi^2) + f_2(z)\boldsymbol{d}_2(\xi^1, \xi^2). \tag{5}$$

In the deformed state, the base vectors are:

$$\boldsymbol{g}_\alpha = \boldsymbol{a}_\alpha + f_1(z)\boldsymbol{d}_{1,\alpha} + f_2(z)\boldsymbol{d}_{2,\alpha}; \quad \boldsymbol{g}_3 = f'_1(z)\boldsymbol{d}_1 + f'_2(z)\boldsymbol{d}_2. \tag{6}$$

The metric tensor components in the deformed configuration C_t are separated into the in-plane and out-of-plane part components. With some approximations, the metric tensor can be written as:

$$g_{ij} = \boldsymbol{g}_i \cdot \boldsymbol{g}_j, \quad \begin{cases} g_{\alpha\beta} \approx a_{\alpha\beta} + f_1(z)b^1_{\alpha\beta} + f_2(z)b^2_{\alpha\beta} \\ g_{\alpha 3} \approx f'_1(z)c^1_\alpha + f'_2(z)c^2_\alpha \\ g_{33} \approx (f'_1 + f'_2)^2 d \end{cases} \tag{7}$$

where $a_{\alpha\beta}, b^k_{\alpha\beta}$ and c^k_α $(k = 1, 2)$ represent the covariant metric surface, the first curvature tensors and the shear, respectively. The parameter d denotes the thickness stretching.

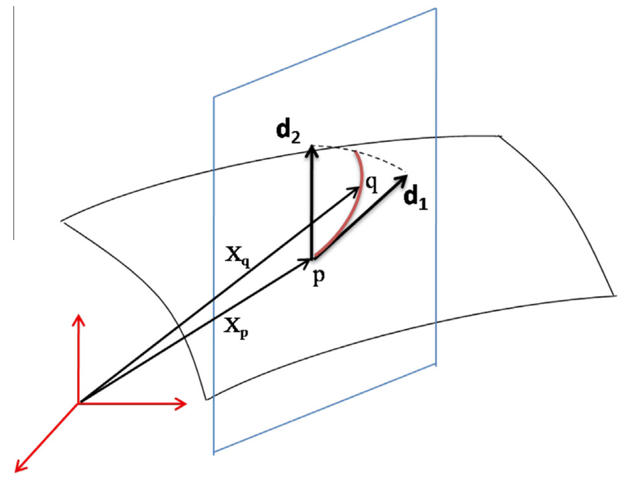

Fig. 1. Double directors shell model.

Taking into account $\boldsymbol{d}_1 \cdot \boldsymbol{d}_1 \approx \boldsymbol{d}_2 \cdot \boldsymbol{d}_2 \approx \boldsymbol{d}_1 \cdot \boldsymbol{d}_2$, these components can be computed as:

$$a_{\alpha\beta} = \boldsymbol{a}_\alpha \cdot \boldsymbol{a}_\beta, \quad c^k_\alpha = \boldsymbol{a}_\alpha \cdot \boldsymbol{d}_k, \quad b^k_{\alpha\beta} = \boldsymbol{a}_\alpha \cdot \boldsymbol{d}_{k,\beta} + \boldsymbol{a}_\beta \cdot \boldsymbol{d}_{k,\alpha},$$
$$d = \boldsymbol{d}_1 \cdot \boldsymbol{d}_1, \quad k = 1, 2. \tag{8}$$

Similar expressions for the in-plane and out-of-plane components of the metric tensor can be obtained in the case of the initial configuration C_0.

Using the kinematic assumption, Eq. (7), the linearized strains can be written as follows [11,12]:

$$\begin{cases} \epsilon_{\alpha\beta} = e_{\alpha\beta} + f_1(z)\chi^1_{\alpha\beta} + f_2(z)\chi^2_{\alpha\beta} \\ 2\epsilon_{\alpha 3} = f'_1(z)\gamma^1_\alpha + f'_2(z)\gamma^2_\alpha \end{cases}, \quad \alpha, \beta = 1, 2. \tag{9}$$

In matrix notation, the vectors of membrane, bending and shear strains are given by:

$$\boldsymbol{e} = \begin{Bmatrix} e_{11} \\ e_{22} \\ 2e_{12} \end{Bmatrix}, \quad \boldsymbol{\chi}^k = \begin{Bmatrix} \chi^k_{11} \\ \chi^k_{22} \\ 2\chi^k_{12} \end{Bmatrix}, \quad \boldsymbol{\gamma}^k = \begin{Bmatrix} \gamma^k_1 \\ \gamma^k_2 \end{Bmatrix}, \quad k = 1, 2. \tag{10}$$

The variation of the strain can be written, in the initial configuration, as:

$$\begin{cases} \delta e_{\alpha\beta} = 1/2(\boldsymbol{A}_\alpha \cdot \delta\boldsymbol{x}_{,\beta} + \boldsymbol{A}_\beta \cdot \delta\boldsymbol{x}_{,\alpha}), \quad \delta\gamma^k_\alpha = \boldsymbol{A}_\alpha \cdot \delta\boldsymbol{d}_k + \delta\boldsymbol{x}_{,\alpha} \cdot \boldsymbol{d}_k \\ \delta\chi^k_{\alpha\beta} = 1/2(\boldsymbol{A}_\alpha \cdot \delta\boldsymbol{d}_{k,\beta} + \boldsymbol{A}_\beta \cdot \delta\boldsymbol{d}_{k,\alpha} + \delta\boldsymbol{x}_{,\alpha} \cdot \boldsymbol{d}_{k,\beta} + \delta\boldsymbol{x}_{,\beta} \cdot \boldsymbol{d}_{k,\alpha}) \end{cases}, \quad k = 1, 2. \tag{11}$$

Or in matrix notation:

$$\delta\boldsymbol{e} = \boldsymbol{B}_m \cdot \delta\boldsymbol{x}, \quad \delta\boldsymbol{\chi}^k = \boldsymbol{B}_{bmk}\delta\boldsymbol{x} + \boldsymbol{B}_{bbk}\delta\boldsymbol{d}_k,$$
$$\delta\boldsymbol{\gamma}^k = \boldsymbol{B}_{smk}\delta\boldsymbol{x} + \boldsymbol{B}_{sbk}\delta\boldsymbol{d}_k, \quad k = 1, 2, \tag{12}$$

where the matrix differential operators, relative to the initial configuration (Linear theory), are given by:

$$\boldsymbol{B}_m = \boldsymbol{B}_{bbk} = \begin{bmatrix} \boldsymbol{A}^T_1 \frac{\partial}{\partial \xi^1} \\ \boldsymbol{A}^T_2 \frac{\partial}{\partial \xi^2} \\ \boldsymbol{A}^T_1 \frac{\partial}{\partial \xi^2} + \boldsymbol{A}^T_2 \frac{\partial}{\partial \xi^1} \end{bmatrix}, \quad \boldsymbol{B}_{bmk} = \begin{bmatrix} \boldsymbol{d}^{0^T}_{k,1} \frac{\partial}{\partial \xi^1} \\ \boldsymbol{d}^{0^T}_{k,2} \frac{\partial}{\partial \xi^2} \\ \boldsymbol{d}^{0^T}_{k,1} \frac{\partial}{\partial \xi^2} + \boldsymbol{d}^{0^T}_{k,2} \frac{\partial}{\partial \xi^1} \end{bmatrix},$$
$$\boldsymbol{B}_{smk} = \begin{bmatrix} \boldsymbol{d}^{0^T}_k \frac{\partial}{\partial \xi^1} \\ \boldsymbol{d}^{0^T}_k \frac{\partial}{\partial \xi^2} \end{bmatrix}, \quad \boldsymbol{B}_{sbk} = \begin{bmatrix} \boldsymbol{A}^T_1 \\ \boldsymbol{A}^T_2 \end{bmatrix}, \quad \boldsymbol{d}^0_k = \boldsymbol{D}, \quad k = 1, 2. \tag{13}$$