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An extended phantom node method study of crack propagation of composites under fatigue loading

Chen Wang [1], Xiwu Xu *

State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, No. 29 Yudao Street, Nanjing 210016, China

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ABSTRACT

In the current study, the traditional phantom node method is extended for crack propagation modeling of composite materials under fatigue loading. A fatigue damage variable, directly related to experimental crack propagation rate, is combined with the static one for interface property degradation, while a bilinear cohesive law considering damage initiation is used for quasi-static cracking modeling. The presented fatigue phantom node method is coded as an user-defined element in ABAQUS and its numerical implementation, including global data, element connectivity and, especially, a pre-cracking treatment, has been discussed in detail in consideration of limitations from the commercial FE software. The proposed method has been validated based on mode I, mode II and mixed mode fatigue crack propagation experiments and, based on that, influences of interface strengths and element size on crack propagation rate evaluation have been investigated briefly.

1. Introduction

High-cycle fatigue is a common cause of failure in aerospace composite structures. For laminated composite materials, failure may involve different processes such as fiber fracture, matrix cracking and interface delamination. Due to mutual interaction between different failure modes, complicated features like specimen size dependence arise, which makes analytical prediction of failure questionable. Therefore, the need for numerical models targeting on detailed failure process increases gradually.

In numerical analysis of failure of composites, matrix cracking and interface delamination are the two most important features. For quasi-static loading, delamination is frequently treated by inserting cohesive elements between plies [1,2]. In 2007, Turon [3] extended the traditional cohesive element method for fatigue delamination modeling of composite laminates by combining a Paris-law related damage variable. After that, several researchers have developed the fatigue cohesive element method and have used this method for delamination propagation rate evaluation under different conditions [4–7]. Unfortunately, although the cohesive element method is effective and efficient for delamination modeling, it is not that promising in intraply matrix cracking study in consideration of the meshing process. For example, Nixon-Pearson [8] has used the cohesive element method in modeling both delamination and matrix cracking of an open-hole composite specimen under fatigue loading. In his study, although simulated failure process agrees well with his experiment, meshing is inconvenient because intraply elements should be placed in ply staking direction in order to make inserted cohesive elements along cracking path. Moreover, predetermination of cracking area is in need for simplifying meshing process, which is, however, not always possible.

For modeling matrix cracking of composites, a more common way is to use continuum damage models in which damages are treated as material state variables in constitutive relationships. Lots of researchers have proposed successful continuum damage models for composites based on some famous failure criteria, such as the criteria of Hashin [9], a more complicated criteria by Puck [10] and the series of LaRC criteria by Camanho [11]. For matrix cracking under fatigue loading, Shokrieh [12], in 2000, extended Hashin's criteria by gradually degrading material stiffness and strengths based on a normalized fatigue life model. Similar to Shokrieh, Kennedy [13] has extended Puck's criteria and Nikishkov [14] has proposed another fatigue damage model based on LaRC criteria and has compared its accuracy with one based on Hashin's.

However, limitation still exists for continuum damage models even though they have been widely used for intraply damage modeling. In studying the performance of continuum models, van der

* Corresponding author at: Mailbox 271, No. 29 Yudao Street, Nanjing 210016, China.

E-mail addresses: wangchen1001@nuaa.edu.cn (C. Wang), xwxu@nuaa.edu.cn (X. Xu).

[1] Postal address: Mailbox 271, No. 29 Yudao Street, Nanjing 210016, China.

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Meer [15] showed that the simulated matrix damage propagation direction gradually deviated from experimental observation in modeling off-axis tension of unidirectional lamina by using a continuum damage model. He believed that it is homogenization and localization inherent in continuum model that hampers the modeling of development of discontinuity and he developed Hansbo's phantom node method (PNM) [16] instead for modeling matrix cracking of composite materials [17]. Different from continuum damage models, in PNM, crack is treated explicitly and crack opening and shearing are calculated based on displacements of both original nodes and phantom nodes added on top of each original one. PNM is also superior to the cohesive element method because of its capability of modeling cracks along almost any direction on a fixed mesh. Moreover, as proved by Song [18], PNM is equivalent to standard XFEM which is also based on the idea of partition of unity method. Due to all those merits, van der Meer [17,19,20] and Yang [21–23] have made a great contribution to the development of PNM and have also achieved a great success in modeling quasi-static failure of composites.

However, to the best of our knowledge, PNM has not been extended to study matrix cracking under fatigue loading till now. Therefore, an attempt is made in this research based on the augmented element method by Ling [21], which is one of the well-known works of PNM. A Paris-law related fatigue damage mechanism similar to that of Turon [3] is introduced into the cohesive law of PNM and effectiveness of the proposed method is verified against a group of frequently used benchmark models including a double cantilever beam (DCB) test, a 4 point End Notch Flexure (4ENF) test, and a Mixed-Mode Bending (MMB) test.

This paper is organized as follows. After this introduction, the basic theory of PNM will be briefed firstly and then numerical implementation of PNM in the commercial FE software ABAQUS will be discussed in detail. After that, in Section 3, the fatigue damage mechanism used will be presented and then verifications will be shown in Section 4. In the final section, main points of the present work will be summarized.

2. Phantom node method

The phantom node method was firstly proposed by Hansbo [16] and has been proved by Ling to be equivalent to the combination of two standard continuum elements and one cohesive element [21]. In this section, the basic idea of PNM will be briefly reviewed and its numerical implementation in the commercial FE software ABAQUS for crack propagation modeling of composite material under quasi-static loading condition will be described in detail.

2.1. Element formulation

The phantom node method treats discontinuity in an explicit way similar to that of XFEM, but with only straight internal crack segments under consideration. When a crack propagates through an element, the element becomes divided into two separate subdomains. In the phantom node method, only cases with either two quadrilateral subdomains or one triangular and one pentagonal subdomains are considered. For both of the two cases, those two subdomains are represented by two overlapping mathematical elements (MEs) which are partially active in the part corresponding to the subdomain they represent.

Taking the quadrilateral case shown in Fig. 1 as an example, when original element cracks, phantom nodes are added on top of real ones and two partially active mathematical elements, referred to as ME1 and ME2, are constructed to represent those two separated subdomains Ω_1 and Ω_2 respectively. Each of the two MEs contains both phantom and real nodes with nodes at the active side being real ones and nodes at the other side being phantom ones. Based on this principle, for the case shown in Fig. 1, element-node connectivity of the two mathematical elements is as follows:

$$\begin{aligned} \text{nodes}_{ME1} &= [1, 2, 3', 4'] \\ \text{nodes}_{ME2} &= [1', 2', 3, 4] \end{aligned} \tag{1}$$

This principle also holds for the case with triangular and pentagonal subdomains, which results in that one ME contains three real nodes as well as one phantom node while the other contains one real node plus three phantom nodes.

Due to the element-node connectivity principle used, the two MEs do not share nodes and, therefore, their displacement fields are independent. For both of the elements, a standard bilinear shape function is used for displacement field interpolation as:

$$\mathbf{u}_i(\mathbf{x}) = \mathbf{N}(\mathbf{x})\mathbf{d}_i, \quad \mathbf{x} \in ME\ i \tag{2}$$

Then under the assumption of small displacement, strain in each ME can be expressed as:

$$\varepsilon_i(\mathbf{x}) = \mathbf{L}\mathbf{u}_i = \mathbf{B}(\mathbf{x})\mathbf{d}_i, \quad \mathbf{x} \in ME\ i \tag{3}$$

where $\mathbf{L}$ is the differential operator under small displacement assumption. Then, stress in each subdomain can be calculated based on linear elasticity.

In the phantom node method, a pair of cohesive tractions is defined at crack faces of the two subdomains for crack propagation modeling as shown in Fig. 1. Displacement jump over the crack is defined as the difference between displacement fields of two MEs at the crack face as:

$$\Delta\mathbf{u}(\mathbf{x}) = \mathbf{u}_1(\mathbf{x}) - \mathbf{u}_2(\mathbf{x}) = \mathbf{N}(\mathbf{x})(\mathbf{d}_1 - \mathbf{d}_2), \quad \mathbf{x} \in \Gamma_C \tag{4}$$

Then, in the crack surface coordinate system, cohesive traction can be determined from the crack face displacement jump according to the cohesive law used as:

$$\bar{\mathbf{t}} = \bar{\mathbf{t}}(\Delta\mathbf{u}) \tag{5}$$

With displacement fields of the two subdomains connected by the cohesive tractions at crack faces, a 8-node phantom node element 1-2-3-4-1′-2′-3′-4′ can be established. Following the standard variational principles, nodal force and element tangent stiffness matrix of the phantom node element can be determined as:

$$\mathbf{f}_1^{\text{int}} = \int_{\Omega_1} \mathbf{B}^T \sigma d\Omega + \int_{\Gamma_C} \mathbf{N}^T \mathbf{t} d\Gamma \tag{6}$$

$$\mathbf{f}_2^{\text{int}} = \int_{\Omega_2} \mathrm{B}^T \sigma d\Omega - \int_{\Gamma_C} \mathbf{N}^T \mathbf{t} d\Gamma \tag{7}$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_1 & \\ & \mathbf{K}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\Delta\mathbf{u}} & -\mathbf{K}_{\Delta\mathbf{u}} \\ -\mathbf{K}_{\Delta\mathbf{u}} & \mathbf{K}_{\Delta\mathbf{u}} \end{bmatrix} \tag{8}$$

$$\mathbf{K}_i = \int_{\Omega_i} \mathbf{B}^T \mathbf{D} \mathbf{B} d\Omega, \quad i = 1, 2 \tag{9}$$

$$\mathbf{K}_{\Delta\mathbf{u}} = \int_{\Gamma_C} \mathbf{N}^T \mathbf{T} \mathbf{N} d\Gamma \tag{10}$$

where bulk material stress σ, cohesive traction $\mathbf{t}$, elastic matrix $\mathbf{D}$ and tangent stiffness matrix of cohesive segment $\mathbf{T}$ are all expressed in the global coordinate system. Shape function $\mathbf{N}$ above is always the standard bilinear one as used before in displacement field interpolation of MEs. Since nodal force vector $\mathbf{f}_i^{\text{int}}$ and stiffness matrix $\mathbf{K}_i$ are integrated only in the active part of each ME, a subdomain integration scheme with three-point Hammer integration on triangular subdomains as shown in Fig. 1 is used instead of the standard Gauss