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Vibration analysis of a specially orthotropic composite laminate with rectangular cutout using independent coordinate coupling method

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ABSTRACT

In this paper, the independent coordinate coupling method is further developed and applied for solving the free vibration of a specially orthotropic composite laminate with rectangular cutout. The total energies of the laminate with cutout are calculated by subtracting the energies of the cutout domain from the energies of the laminate domain, where the energies of laminate domain and cutout domain are calculated independently. The mode shape functions of beams with various boundary conditions are considered in each direction as admissible functions. Two coordinate systems are coupled by imposing the displacement matching condition at the cutout domain. So it is very convenient to construct and calculate the mass and stiffness matrices. The effects of modulus ratio, aspect ratio and boundary conditions on the natural frequencies are investigated. The numerical results demonstrate that the proposed methodology can be used as an efficient tool to solve the free vibration problems of specially orthotropic composite laminates with rectangular cutout.

1. Introduction

With the fast development of material science and technology, fiber reinforced composite materials [1], which have considerable advantages over traditional metal and nonmetal materials, have received widespread attentions since last decades. Such kind of composite materials usually consist of many individual layers which have different fiber orientations. Therefore, they can sustain large mechanical force from various directions. In many cases, the composite laminates or panels are required to open holes [2,3] to meet the design requirements, such as bolt and pipe connections, failure inspection, and even weight reduction. Since opening holes changes the structural weight and stiffness, we have to analyze the dynamic characteristics of plates and shells with the existence of holes to avoid the resonance of structures.

The vibration of plate with rectangular or circular hole has been widely studied by either the Rayleigh–Ritz method or the finite element method. Ali [4] presented a Rayleigh–Ritz method for the dynamic analysis of rectangular plates with rectangular cutouts. Lam [5] studied the vibrations of plates with stiffened openings using the partitioning method and orthogonal polynomials as admissible functions. He divided the plate into sub-domains and then applied the Rayleigh–Ritz method. Paramasivam [6] analyzed the free vibration problem of rectangular plate with square openings for different types of boundary conditions by using the finite difference method. The vibration characteristics of isotropic plate with circular holes has also been comprehensively studied [7]. However, since the geometry of hole is circular, it leads to more complicate calculations for the vibration analysis. Hegarty [8] developed the least-squares point-matching method for the free vibrations of a rectangular elastic plate with a central circular hole. His method employed the polar coordinate for the hole domain and the global plate. Lee and Lim [9] also used the point-matching technique and Rayleigh method to study the free vibrations of isotropic and orthotropic square plates with cutouts subjected to in-plane forces through sub-dividing the plate into sub-domains. Liew et al. [10] used the discrete Ritz method to study the free vibration of rectangular plates with central cutouts. The approach is belong to the domain decomposition method [11]. Kwak [12] recently developed an independent coordinate coupling method for analyzing an isotropic plate with rectangular and circular holes. His approach is computationally very convenient and efficient to calculate the integrations.

The vibration analysis of composite laminates with cutout has also received considerable attentions since the last decade.

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However, due to their layer independent material constants, the modeling seems to be more complicate. Rajamani et al. [13,14] formulated the forced and free vibrations of composite plates with cutouts. They assumed that the cutout is equivalent to an external loading on the plate and studied the laminate with simply-supported boundary condition and symmetric stacking sequence. Boay [15] studied the vibration problem of symmetrically laminated composite plates with circular hole by means of the finite element method. He studied the hole diameter effect on the natural frequencies. Pandit et al. [16] also used a finite element method to analyze the free vibration of fiber reinforced laminated composite plates with cutouts based on the first order shear deformation theory. They inspected the effects of fiber orientation angles, thickness ratios and aspect ratios on the natural frequencies. Nallim et al. [17] studied the natural frequencies of symmetrically laminated elliptical and circular plates based on the Rayleigh–Ritz method where the deflection is approximated by the polynomial functions. Nanda [18] investigated the nonlinear free vibration of laminated composite cylindrical shells with cutouts based on the finite element method using an eight-node C^0 continuity, isoparametric quadrilateral element. Natarajan et al. [19] investigated the free vibration and buckling of laminated composites with cutouts, taking into the consideration of hygrothermal effects. The analysis was carried out based on the extended finite element method.

In this paper, we investigate the free vibration of a specially orthotropic laminate with central rectangular cutout by using the independent coordinate coupling method. The proposed approach is based on the Rayleigh–Ritz method, but adopts two coordinate systems for the laminate domain and cutout domain, respectively. In the present method, the total energies are calculated by subtracting the energies of the cutout domain from the laminate domain. By the Lagrange's equation, the equation of motion can be obtained. The mode shape functions of beam with various boundary conditions are adopted as admissible functions in each direction. To couple the two independent coordinate systems, the displacement matching condition is applied at the cutout domain that provides the kinematic relation between two coordinate systems. With the proposed method, several examples are studied that considers the effects of layup stacking sequence, modulus ratio, aspect ratio and boundary condition. It is found that the proposed method is very convenient and efficient in solving the vibration problem of symmetrically laminated composites with rectangular cutout.

2. Formulations of the problem

A rectangular laminated plate with rectangular cutout is shown in Fig. 1. It can been seen that two coordinate systems, (x, y, z) and (x_h, y_h, z_h), are used for the global perfect laminate domain and the cutout domain individually in the present independent coordinate coupling method. The plate has the width a in the x direction, the length b in the y direction and the height h in the z direction, and the cutout has the width a_h in the x_h direction and length b_h in the y_h direction. The laminate consists of several orthotropic layers with symmetric layup configurations. The stress–strain relation for the kth lamina is given by

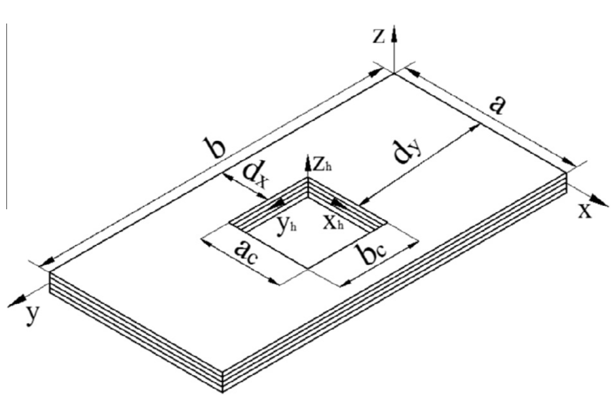

Fig. 1. Coordinate system of composite laminate with a rectangular cutout.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}_{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & Q_{16} \\ & Q_{22} & 0 & 0 & Q_{26} \\ & & Q_{44} & Q_{45} & 0 \\ & symm. & & Q_{55} & 0 \\ & & & & Q_{66} \end{bmatrix}_{(k)} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \tag{1}$$

where $Q_{ij}^{(k)}$ are the material constants of the kth lamina in the laminate coordinate system.

The kinematic and potential energies are expressed in two coordinate systems for the laminate and the cutout domains independently. It has the following expressions for the laminate domain:

$$T = \frac{1}{2}\rho h \int_0^a \int_0^b \dot{w}(x, y, t)^2 dxdy \tag{2a}$$

$$V = \frac{1}{2}\int_0^a \int_0^b \left(D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 + D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 \right) dxdy \tag{2b}$$

and for the cutout domain:

$$T_h = \frac{1}{2}\rho h \int_0^{a_h} \int_0^{b_h} \dot{w}_h(x_h, y_h, t)^2 dx_h dy_h \tag{2c}$$

$$V_h = \frac{1}{2}\int_0^{a_h} \int_0^{b_h} \left(D_{11}\left(\frac{\partial^2 w_h}{\partial x_h^2}\right)^2 + 2D_{12}\frac{\partial^2 w_h}{\partial x_h^2}\frac{\partial^2 w_h}{\partial y_h^2} + 4D_{66}\left(\frac{\partial^2 w_h}{\partial x_h \partial y_h}\right)^2 + D_{22}\left(\frac{\partial^2 w_h}{\partial y_h^2}\right)^2 \right) dx_h dy_h \tag{2d}$$

where $w(x, y, z, t)$ and $w_h(x, y, z, t)$ represent the deflections of the laminate and cutout, ρ is the density, and D_{ij} are the bending stiffness components which have the following expressions.

$$D_{ij} = \int_{-h/2}^{h/2} \bar{Q}_{ij} z^2 dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)} z^2 dz = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{ij}^{(k)} (z_{k+1}^3 - z_k^3) \tag{3}$$

Considering the generality and convenience, the nondimensional coordinates are defined for both coordinates as

$$\xi = x/a, \quad \eta = y/b \tag{4a}$$

$$\xi_h = x_c/a_c, \quad \eta_h = y_c/b_c \tag{4b}$$

Therefore, by means of the assumed mode method, the deflections of the laminate and the cutout can be expressed as

$$w(\xi, \eta, t) = \Phi(\xi, \eta) q(t) \tag{5a}$$

$$w_h(\xi_h, \eta_h, t) = \Phi_h(\xi_h, \eta_h) q_h(t) \tag{5b}$$

where the admissible functions Φ and Φ_h consist of n and n_h terms, and the generalized coordinates have the following matrix forms for the laminate and cutout.

$$\Phi(\xi, \eta) = [\Phi_1 \quad \Phi_2 \quad \cdots \quad \Phi_n]_{1\times n}, \quad q(t) = [q_1 \quad q_2 \quad \cdots \quad q_n]_{n\times 1}^T \tag{6a, b}$$