Composite Structures 150 (2016) 103–114

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Analysis of laminated composites and sandwich structures by trigonometric, exponential and miscellaneous polynomials and a MITC9 plate element

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ARTICLE INFO

Article history:
Received 13 October 2015
Revised 11 December 2015
Accepted 28 December 2015
Available online 1 April 2016

Keywords:
Trigonometric
Exponential
Finite Element Method
Mixed Interpolation of Tensorial Components
Carrera Unified Formulation, Plate

ABSTRACT

This paper proposes some advanced plate theories obtained by expanding the unknown displacement variables along the thickness direction using trigonometric series, exponential functions and miscellaneous polynomials. The used refined models are Equivalent Single Layer (ESL) theories. They are obtained by means of the Unified Formulation by Carrera (CUF), and they accurately describe the displacement field and the stress distributions along the thickness of the multilayered plate. The governing equations are derived from the Principle of Virtual Displacement (PVD), and the Finite Element Method (FEM) is employed to solve them. The plate element has nine nodes, and the Mixed Interpolation of Tensorial Components (MITC) method is used to contrast the membrane and shear locking phenomenon. Cross-ply plates with simply-supported edges and subjected to a bi-sinusoidal load, and sandwich plates with simply-supported edges and subjected to a constant transverse uniform pressure are analyzed. Various thickness ratios are considered. The results, obtained with different theories within CUF, are compared with the elasticity solutions given in the literature and the layer-wise solution. It is shown that refined kinematic theories employing trigonometric or exponential terms are able to accurately describe the displacement field and the mechanical stress fields. In some cases, the reduction of computational costs is particularly relevant respect to the layer-wise solution.

1. Introduction

Composite plate/shell structures have a predominant role in many engineering applications. Structural models for composite plates must be able to deal with a number of physical effects such as anisotropy, shear deformation and interlaminar continuity of shear stress. Analytical, closed form solutions are available in very few cases. In most of the practical problems, the solution demands applications of approximated computational methods. The Finite Element Method (FEM) has a predominant role among the computational techniques implemented for the analysis of layered structures. Finite elements are usually formulated on the basis of axiomatic-type theories, in which the unknown variables are postulated along the thickness. According to published research, various theories for composite structures have been developed. They can be classified as: Equivalent Single Layer (ESL), in which the number of unknowns is independent of the number of layers, and Layer-wise approach (LW), in which the number of unknowns is dependent on the number of layers. The simplest plate/shell theory is based on the Kirchoff/Love's hypothesis, and it is usually referred to as Classical Lamination Theory (CLT) [1,2]. The inclusion of transverse shear strains leads to the Reissner–Mindlin Theory, also known as First-order Shear Deformation Theory (FSDT) [3]. A review of Equivalent Single Layer and layer-wise laminate theories was presented by Reddy [4]. Also, a large variety of plate/shell finite element implementations of higher-order theories (HOT) has been proposed in the last twenty years. HOT-type theories were discussed by Kant and co-authors [5,6], by Reddy [7] and Palazotto and Dennis [8].

Concerning trigonometric polynomial expansions, some plate and beam theories have been developed. Shimpi and Ghugal [9] used trigonometric terms in the displacements field for the analysis of two layers composite beams. An ESL model was developed by Arya et al. [10] using a sine term to represent the non-linear displacement field across the thickness in symmetrically laminated beams. An extension of [10] to composite plates was presented by Ferreira et al. [11]. A trigonometric shear deformation theory is used to model symmetric composite plates discretized by a

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http://dx.doi.org/10.1016/j.compstruct.2015.12.038
0263-8223/

meshless method based on global multiquadric radial basis functions. A version of this theory, with a layer-wise approach, was proposed by the same authors in [12]. Vidal and Polit [13] developed a new three-noded beam finite element for the analysis of laminated beams, based on a sine distribution with layer refinement. Recently, the same authors have dealt with the influence of the Murakami's zig–zag function in the sine model for static and vibration analysis of laminated beams [14]. Static and free vibration analysis of laminated shells were performed by radial basis functions collocation, according to a sinusoidal shear deformation theory in Ferreira et al. [15]. It accounts for through-the-thickness deformation, by considering a sinusoidal evolution of all displacements along the thickness coordinate. The complexity of some structures often requires the adoption of 3D numerical solutions to describe the mechanical behavior properly. The main limitation of the 3D finite elements, which are implemented in the commercial codes, is the significant computational cost when large-scale structures are considered. Although advanced 3D finite elements have been recently proposed to lessen this issue [16], refined 1D and 2D elements still represent valuable tools for reducing the computational effort.

In this work, an improved plate finite element is presented for the analysis of plate multilayered structures. It is based on the Carrera Unified Formulation (CUF), which was developed by Carrera for multi-layered structures [17,18]. Within the CUF framework, several beam models using trigonometric, exponential, hyperbolic and miscellaneous series were employed [19,20]. A review of Equivalent Single Layer and layer-wise laminate theories was presented in [21]. In the present work, a number of advanced ESL plate theories, obtained by use of Taylor polynomials, trigonometric series, and exponential functions, are discussed. The Mixed Interpolation of Tensorial Components (MITC) method [22–24] is used to contrast the membrane and shear locking. The governing equations in weak form for the linear static analysis of composite structures are derived from the Principle of Virtual Displacement (PVD), and the Finite Element Method is used to solve them. Cross-ply plates with simply-supported edges and subjected to a bi-sinusoidal load, and sandwich plates with simply-supported edges and subjected to a constant transverse uniform pressure are analyzed. The results, obtained with the different models, are compared with both exact solutions and higher-order theories solutions given in literature.

This paper is organized as follows: geometrical and constitutive relations for plates are presented in Section 2. In Section 3, an overview of classical, higher-order and advanced plate theories developed within the CUF framework is given. Section 4 gives a brief outline of the FEM approach and the MITC9 method to overcome the problem of shear locking, whereas, in Section 5, the governing equations in weak form for the linear static analysis of composite structures are derived from the PVD. In Section 6, the results obtained using the proposed CUF theories are discussed. Section 7 is devoted to the conclusions.

2. Geometrical and constitutive relations for plates

Plates are bi-dimensional structures in which one dimension (in general the thickness in the z direction) is negligible with respect to the other two in-plane dimensions. The geometry and the reference system are indicated in Fig. 1. Geometrical relations enable to express the in-plane ϵ_p^k and out-plane ϵ_n^k strains in terms of the displacement $\boldsymbol{u}$:

$$\boldsymbol{\epsilon}_p^k = [\epsilon_{xx}^k, \epsilon_{yy}^k, \epsilon_{xy}^k]^T = (\boldsymbol{D}_p^k)\,\boldsymbol{u}^k, \quad \boldsymbol{\epsilon}_n^k = [\epsilon_{xz}^k, \epsilon_{yz}^k, \epsilon_{zz}^k]^T = (\boldsymbol{D}_{np}^k + \boldsymbol{D}_{nz}^k)\,\boldsymbol{u}^k. \tag{1}$$

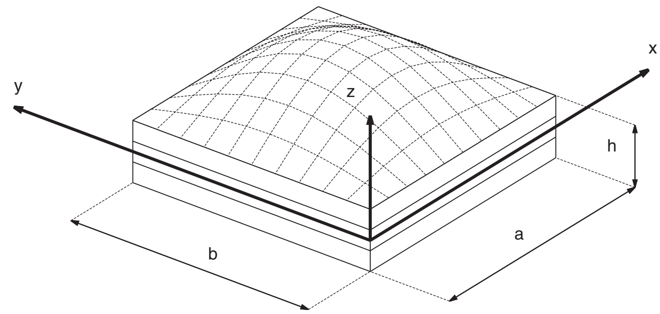

Fig. 1. Reference system of the plate with a bi-sinusoidal loading.

The explicit form of the introduced arrays of the differential operators is:

$$\boldsymbol{D}_p^k = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ \partial_y & \partial_x & 0 \end{bmatrix}, \quad \boldsymbol{D}_{np}^k = \begin{bmatrix} 0 & 0 & \partial_x \\ 0 & 0 & \partial_y \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{D}_{nz}^k = \begin{bmatrix} \partial_z & 0 & 0 \\ 0 & \partial_z & 0 \\ 0 & 0 & \partial_z \end{bmatrix}. \tag{2}$$

The stress–strain relations are:

$$\begin{aligned} \boldsymbol{\sigma}_p^k &= \boldsymbol{C}_{pp}^k\,\boldsymbol{\epsilon}_p^k + \boldsymbol{C}_{pn}^k\,\boldsymbol{\epsilon}_n^k, \\ \boldsymbol{\sigma}_n^k &= \boldsymbol{C}_{np}^k\,\boldsymbol{\epsilon}_p^k + \boldsymbol{C}_{nn}^k\,\boldsymbol{\epsilon}_n^k, \end{aligned} \tag{3}$$

where

$$\begin{aligned} \boldsymbol{C}_{pp}^k &= \begin{bmatrix} C_{11}^k & C_{12}^k & C_{16}^k \\ C_{12}^k & C_{22}^k & C_{26}^k \\ C_{16}^k & C_{26}^k & C_{66}^k \end{bmatrix} \quad \boldsymbol{C}_{pn}^k = \begin{bmatrix} 0 & 0 & C_{13}^k \\ 0 & 0 & C_{23}^k \\ 0 & 0 & C_{36}^k \end{bmatrix}, \\ \boldsymbol{C}_{np}^k &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_{13}^k & C_{23}^k & C_{36}^k \end{bmatrix} \quad \boldsymbol{C}_{nn}^k = \begin{bmatrix} C_{55}^k & C_{45}^k & 0 \\ C_{45}^k & C_{44}^k & 0 \\ 0 & 0 & C_{33}^k \end{bmatrix}. \end{aligned} \tag{4}$$

For the sake of brevity, the expressions, that relate the material coefficients C_{ij} to the Young's moduli E_1, E_2, E_3, the shear moduli G_{12}, G_{13}, G_{23} and Poisson moduli $\nu_{12}, \nu_{13}, \nu_{23}, \nu_{21}, \nu_{31}, \nu_{32}$ that characterize the layer material, are not given here. They can be found in [4].

3. Carrera Unified Formulation for plates

According to the CUF [18,25,26], the displacement field can be written as follows:

$$\begin{cases} u(x,y,z) = F_0(z)\,u_0(x,y) + F_1(z)\,u_1(x,y) + \cdots + F_N(z)\,u_N(x,y), \\ v(x,y,z) = F_0(z)\,v_0(x,y) + F_1(z)\,v_1(x,y) + \cdots + F_N(z)\,v_N(x,y), \\ w(x,y,z) = F_0(z)\,w_0(x,y) + F_1(z)\,w_1(x,y) + \cdots + F_N(z)\,w_N(x,y). \end{cases} \tag{5}$$

In compact form:

$$\boldsymbol{u}^k(x,y,z) = F_s(z)\boldsymbol{u}_s^k(x,y); \quad \delta\boldsymbol{u}^k(x,y,z) = F_\tau(z)\delta\boldsymbol{u}_\tau^k(x,y) \\ \tau, s = 0, 1, \ldots, N, \tag{6}$$

where (x,y,z) is the general reference system, see Fig. 1, and the displacement vector $\boldsymbol{u} = \{u, v, w\}$ has its components expressed in this system. $\delta\boldsymbol{u}$ is the virtual displacement associated to the virtual work and k identifies the layer. F_τ and F_s are the thickness functions depending only on z. $\boldsymbol{u}_s$ are the unknown variables depending on the coordinates x and y. τ and s are sum indexes and N is the number of terms of the expansion in the thickness direction assumed for the displacements.