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A series–parallel mixture model to predict the overall property of particle reinforced composites

J.F. Tan [a], Y.J. Jia [b], L.X. Li [a,*]

[a] State Key Laboratory for Strength and Vibration of Mechanical Structures, School of Aerospace, Xi'an Jiaotong University, Xi'an 710049, PR China
[b] The 41st Institute of the Fourth Academy of CASC, Xi'an 710025, PR China

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ABSTRACT

Motivated by stress and strain distributions in the unit cell obtained by the finite element analysis, a series–parallel mixture model is proposed to predict the overall property of particle reinforced composites. In this model, the matrix (softer) phase is split to two parts. The particle is first connected to one part in series, and then connected to the remaining part in parallel, eventually to form the composite. Numerical studies show that the in-series ratio is independent of material parameters and can be fitted as a function of particle volume fraction. The model is finally applied to predicting the overall property of composites with more than two phases and the relaxation of viscoelastic composites. The results validate the accuracy of the model.

1. Introduction

There have been numbers of estimates to predict overall properties of composites. The two extreme rule-of-mixture models are the Voigt Estimate (VE) and Reuss Estimate (RE) that are proposed to describe two-phase composites [1]. The VE corresponds to the case when the applied load causes equal strains in the two phases. The overall composite stress is the sum of stresses carried by each phase. Therefore, the overall Young's modulus is the average of the moduli of the constituents weighted by the volume fraction of each phase. The RE corresponds to the case when each phase of the composite carries an equal stress. The overall strain in the composite is the sum of the net strain carried by each phase, and the overall compliance is the average of the compliance of the constituents weighted by the volume fraction of each phase. Hill [2] has shown that VE and RE are respectively the upper and lower bounds of the true overall elastic modulus. Because the two estimates cannot reflect the detailed constituent geometry, the dispersion structure and so on, its accuracy is highly questionable [3].

The Mori-Tanaka Estimate (MTE) and the Self-Consistent Estimate (SCE) are the two somewhat complex but practical estimates. For a particle-reinforced composite [4], the MTE of the overall bulk and shear moduli was derived by Benveniste [5] by taking the matrix to be the homogenized comparison material (HCM). In the SCE proposed by Hill [2], Budiansky [6] and Hori and Nemat-Nasser [7], the HCM was the overall material under study. The SCE seems more reliable [8], but sometimes fails to accurately predict the overall properties, or even violates a certain upper bound at a small fiber volume fraction [9].

In order to predict the elastic modulus of two phase materials more accurately, Tamura and coworkers [10] proposed the Modified Rule of Mixtures (MRM) by introducing an empirical parameter, which was subsequently adopted by Williamson et al. [11], Giannakopulos et al. [12], Kesler et al. [13] and Ku et al. [14]. The MRM was later developed by the current author [15] to be a unified estimate including the MTE and the SCE.

Despite of the fact that a lot of approaches have been developed, the MTE and SCE are the two most popular ones and extensively in use in recent years. For example, by using the MTE, vibrational behavior of continuously graded carbon nanotube-reinforced cylindrical panels [16], single walled carbon nanotube reinforced polymers [17], composites reinforced with short carbon fibers and radially aligned carbon nanotubes [18], "fuzzy fiber" composites [19] and two-phase random composite materials [20] were respectively studied. By using the SCE, the effective stiffness of mixtures of two isotropic phases with isotropic, transversely isotropic, hexahedral and octahedral interface orientation distributions was studied [21].

However, the overall property by the two approaches is still not satisfactory, especially for larger volume fractions. Fig. 1 is the comparison of the MTE and the SCE when $E_p/E_m = 10$ as compared

* Corresponding author.
E-mail address: luxianli@mail.xjtu.edu.cn (L.X. Li).

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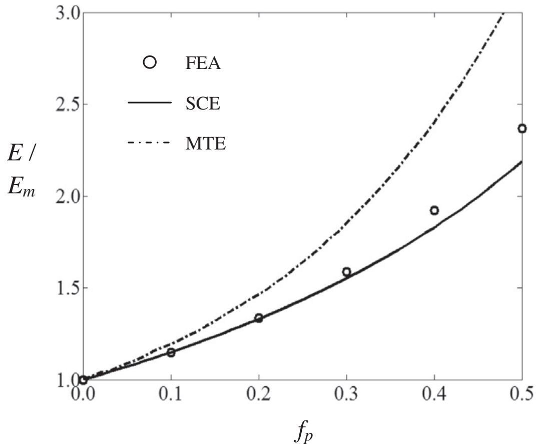

Fig. 1. Overall moduli estimated by the MTE and the SCE. (E_p, E_m, E: modulus of particle, matrix and the overall composite, respectively; f_p: volume fraction of particle).

with the numerical result through the finite element analysis (FEA), which indicates that there is still the room to improve these approaches.

Motivated by stress and strain distributions obtained by the FEA, a series–parallel mixture (SP) model is proposed and validated in this paper. To this end, the paper is outlined as follows. In Section 2, the basic theories of the in-parallel model and the in-series model are introduced respectively for elastic composites and viscoelastic composites. In Section 3, the SP model is proposed through the FEA, and the overall property is then derived together with fitting the in-series ratio. In Section 4, extension of the SP model is studied to composites with more than two phases and with viscoelastic phases. Conclusions are finally made in Section 5 to close this paper.

2. Basic theories

2.1. Simple mixture models for elastic materials

2.1.1. The in-parallel model for elastic materials

As shown in Fig. 2, the two elastic materials are connected together in parallel to form a composite. Assume that the composite is subjected to a uniform tension at the right end, so, we have

$$\varepsilon = \varepsilon_1 = \varepsilon_2 \quad (1)$$

and

$$\sigma = \sigma_1 f_1 + \sigma_2 f_2 \quad (2)$$

where σ and ε are respectively the overall stress and strain. σ_1 and σ_2 are respectively the stresses of matrix phase and particle phase while ε_1 and ε_2 are the strains and f_1 and f_2 are the volume fractions.

As often, with the definition of the overall modulus as

$$E = \sigma / \varepsilon \quad (3)$$

we obtain

$$E = E_1 f_1 + E_2 f_2 \quad (4)$$

for the in-parallel model.

The in-parallel model is also termed as the VE [15].

2.1.2. The in-series model for elastic materials

As shown in Fig. 3, if the two elastic materials are connected together in series, and subjected to a uniform traction at the right end, we have

$$\sigma = \sigma_1 = \sigma_2 \quad (5)$$

and

$$\varepsilon = \varepsilon_1 f_1 + \varepsilon_2 f_2 \quad (6)$$

Thus, we obtain the overall modulus as

$$\frac{1}{E} = \frac{f_1}{E_1} + \frac{f_2}{E_2} \quad (7)$$

The in-series model is also termed as the RE [15].

2.2. Simple mixture models for viscoelastic materials

A viscoelastic material is assumed to be the combination of elastic units and dashpots. One simple viscoelastic material is Maxwell model in which an elastic unit is in series to a dashpot as shown in Fig. 4.

Subjected to a step strain $\varepsilon(t) = \varepsilon_0 H(t)$, the relaxation stress is obtained as

$$\sigma(t) = E(t)\varepsilon_0 \quad (8)$$

with $E(t)$, the relaxation modulus, is

$$E(t) = E_e e^{-\frac{t}{\eta/E_e}} \quad (9)$$

where E_e is the modulus of the elastic unit and η is the viscosity coefficient of the dashpot.

In this context, the viscoelastic material means the material expressed by Maxwell model with the relaxation modulus by Eq. (9).

2.2.1. The in-parallel model for viscoelastic materials

As shown in Fig. 5, the two viscoelastic materials are connected in parallel and subjected to a uniform tension at the right end. According to the parallel rule expressed by Eqs. (1)–(3), together with Eqs. (8) and (9), we have

$$E(t) = f_1 E_1(t) + f_2 E_2(t) \quad (10)$$

with

$$\begin{cases} E_1(t) = E_{1e} e^{-\frac{t}{\eta_1/E_{1e}}} \\ E_2(t) = E_{2e} e^{-\frac{t}{\eta_2/E_{2e}}} \end{cases} \quad (11)$$

If $\eta_1 \to \infty$, for example, the first viscoelastic material will reduce to an elastic material. So, Eqs. (10) and (11) are also applicable for the case of an elastic material connected in-parallel to a viscoelastic material and for the case of both elastic materials connected in-parallel as expressed in Eq. (4).

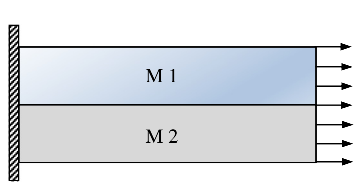

Fig. 2. The in-parallel model.

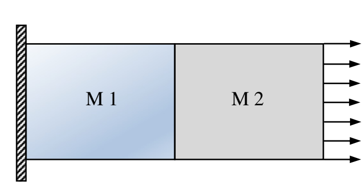

Fig. 3. The in-series model.