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High-order plate finite elements for smart structure analysis

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ABSTRACT

This paper presents new finite elements for plate analysis of smart composite structures. Based on the sinus model, Murakami's Zig–Zag functions are introduced in the three directions, improving the accuracy for multilayered modeling. The transverse normal stress is included allowing use of the three-dimensional constitutive law. Three different eight-node finite elements are developed using C^0 approximations, each with a different number of unknown functions: 9, 11 or 12. For the piezoelectric approximation, a layer wise description is used with a cubic variation in the thickness of each layer while the potential is assumed to be constant on each elementary domain for the in-plane variation. These finite elements aims at modeling both thin and thick plates without any pathologies of the classical plate finite elements (shear and Poisson or thickness locking, spurious modes, etc.). This family is evaluated on classical piezoelectric problems of the literature and special emphasis is pointed towards the introduction of equipotential conditions.

1. Introduction

Research and development concerning high-performance structures are very intense since some decades. Structural health monitoring, active vibration damping, and energy harvesting are some examples of possible applications of a multifunctional structural component. Piezoelectric materials permit to convert mechanical and electrical energy at frequency ranges that are most interesting for technical applications such as vibration damping and rapid shape adaptation [1]. Development of theoretical and numerical models for this kind of structures is very important and active. For this purpose and in the framework of two-dimensional plate/shell models, different choices can be made for the mechanical approximation and the following classification is classically admitted for the variation in the thickness direction: (i) Equivalent Single Layer (ESL) models, in which the number of unknowns is independent of the layer number; (ii) Layer-Wise (LW) descriptions, for which the number of unknowns and, thus, the computational cost increases with the number of layers. While most developments employ an ESL description for the mechanical behavior, and particularly the First order Shear Deformation Theory (FSDT), a Layer-Wise description is necessary for the piezoelectric approximation to impose electric boundary conditions at each piezoelectric layer interfaces, i.e., the electrodes, within the stack. Inside each piezoelectric layer, the electric potential can be linear, quadratic or higher and a comparison has been proposed in [2].

A review of different approaches is available in [3,4] and in the framework of the Carrera Unified Formulation (CUF) in [5]. For the FE approximations, a recent review limited to shell models is also given in [6].

The limitation of the FSDT model is related to the constant transverse displacement hypothesis, inducing no thickness change and the use of the reduced 2D constitutive law. The use of the full 3D constitutive law is an important feature for a consistent representation of complex physical interactions like multi-field coupling. Furthermore, accurate modeling of thick structures needs the transverse normal stress and the 3D constitutive law.

Therefore, a high-order model is chosen with sinus function for the in-plane displacements and quadratic assumption along the thickness of the transverse deflection. Thus, the 3D constitutive law is retained and a parabolic distribution of the transverse shear strains and a non linear variation of the transverse normal strain are recovered. In order to introduce transverse strain discontinuities required to fulfill the interlaminar equilibrium, Murakami's Zig–Zag function (MZZF) [7] is superimposed to the high-order ESL kinematics for the 3 displacement components. Note that MZZF does not depend on the constitutive coefficients and is, hence, attractively simple in conjunction with three-dimensional constitutive laws including multi-field coupling. Based on this kinematics, an 8-node plate finite element (FE) is proposed, free of numerical illness such as transverse shear and Poisson lockings, oscillation and spurious mechanics [8]. The approximation of the

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electric potential must be able to model piezoelectric patches, and a constant value is considered on each elementary domain while a cubic variation in each layer is used, based on the polynomial expansion given in [9,10].

The paper is organized as follows: Section 2 describes the plate problem and the FE approximations are given in Section 3. The resulting FE are validated in Section 4 by referring to well-known linear static piezoelectric and composite plate problems. Finally, Section 5 summarizes the main findings.

2. Description of the plate problem

2.1. Governing equations

Let us consider a plate occupying the domain $\mathcal{V} = \Omega \times \left[-\frac{e}{2} \leqslant z \leqslant \frac{e}{2}\right]$ in a Cartesian coordinate system $(x_1, x_2, x_3 = z)$. The plate is defined by an arbitrary surface Ω in the (x_1, x_2) plane, located at the midplane for $z = 0$, and by a constant thickness e.

The displacement is denoted $\vec{u}(x_1, x_2, z)$ and the electric potential is $\phi(x_1, x_2, z)$. $\varepsilon_{ij}(x_1, x_2, z)$ and $\vec{E}(x_1, x_2, z)$ are the strain tensor components and the electric field vector, respectively, deduced from primal variables by the geometric relations. Furthermore, $\sigma_{ij}(x_1, x_2, z)$ and $\vec{D}(x_1, x_2, z)$ are the conjugated fluxes (stress tensor components and dielectric displacement vector, respectively) obtained from the constitutive equations given in the next subsection.

2.1.1. Constitutive relation

The 3D constitutive equation for a linear piezoelectric material is given by the following set of coupled equations [11] for a layer (k):

$$[\sigma^{(k)}] = [C^{(k)}]\,[\varepsilon^{(k)}] - [e^{(k)}]^T\,[E^{(k)}] \tag{1a}$$

$$[D^{(k)}] = [e^{(k)}]\,[\varepsilon^{(k)}] + [\epsilon^{(k)}]\,[E^{(k)}] \tag{1b}$$

where we denote by $[C]$ the matrix of elastic stiffness coefficients taken at constant electric field, by $[e]$ the matrix of piezoelectric stress coefficients and by $[\epsilon]$ the matrix of electric permittivity coefficients taken at constant strain. The explicit form of these matrices can be found in [6] for an orthotropic piezoelectric layer polarized along the thickness direction z. Eq. (1a) expresses the piezoelectric *converse* effect for actuator applications, whereas Eq. (1b) represents the piezoelectric *direct* effect which is exploited in sensor applications. Note that the constitutive law is expressed in the local reference frame associated to each layer.

2.1.2. The weak form of the boundary value problem

The classical piezoelectric variational formulation of [12] is employed in which the primary field variables are the "generalized displacements", i.e., the displacement field and the electrostatic potential. Using a matrix notation and for admissible virtual displacements $\vec{u}^*$ and electric potential ϕ^* (virtual quantities are denoted by an asterisk), the variational principle is given by:

$$\begin{aligned}\int_{\mathcal{V}} \rho [u^*]^T [\ddot{u}]\, d\mathcal{V} = &- \int_{\mathcal{V}} [\varepsilon(u^*)]^T [\sigma(u,\phi)]\, d\mathcal{V} + \int_{\mathcal{V}} [u^*]^T [f]\, d\mathcal{V} \\ &+ \int_{\partial\mathcal{V}_F} [u^*]^T [F]\, d\partial\mathcal{V} + \int_{\mathcal{V}} [E(\phi^*)]^T [D(u,\phi)]\, d\mathcal{V} \\ &- \int_{\mathcal{V}} q\, \phi^*\, d\mathcal{V} - \int_{\partial\mathcal{V}_Q} Q\, \phi^*\, d\partial\mathcal{V}\end{aligned} \tag{2}$$

where $[f]$ is the body force vector, $[F]$ the surface force vector applied on $\partial\mathcal{V}_F, q$ the volume charge density, Q the surface charge density supplied on ∂C_q and ρ is the mass density. Finally, $\varepsilon(u^*)$ and $E(\phi^*)$ are the virtual strain and virtual electric field that satisfy the compatibility gradient equations. In the remainder of this article we will refer only to static problems, for which the left-hand side term is set to zero. Furthermore, body forces and volume charge densities will be discarded ($[f] = [0]$; $q = 0$).

2.2. The mechanical part

2.2.1. The displacement field

Based on the sinus model, see [13], a new plate model which takes into account the transverse normal stress is presented in this section. This extension is based on following developments:

- various models for beams, plates and shells based on the refined sinus theory, see [13–20];
- our previous paper on a 7 parameter model for thermo-mechanical analysis [8].

In the framework of ESL approach, the kinematics of our model is assumed to have the following particular form

$$\begin{cases} U_1(x_\alpha, z) = u_1^0(x_\alpha) + z\, u_1^1(x_\alpha) + f(z)\, u_1^f(x_\alpha) \\ U_2(x_\alpha, z) = u_2^0(x_\alpha) + z\, u_2^1(x_\alpha) + f(z)\, u_2^f(x_\alpha) \\ U_3(x_\alpha, z) = u_3^0(x_\alpha) + z\, u_3^1(x_\alpha) + z^2\, u_3^2(x_\alpha) \end{cases} \tag{3}$$

where $\alpha \in \{1, 2\}$ and $i \in \{1, 2, 3\}$. In Eq. (3), the superscript is associated to the expansion order in z while the subscript is related to the component of the displacement. Thus, u_i^0 are the displacements of a point of the reference surface while (u_α^1, u_α^f) are measures for rotations of the normal transverse fiber about the axis $(0, x_\alpha)$. The functions u_3^α permit to have a non-constant deflection for the transverse fiber and allow to have non zero transverse normal stretch. Furthermore, the quadratic assumption for the transverse displacement avoids the occurrence of Poisson (or thickness) locking, see [8].

In the context of the sinus model, we have

$$f(z) = \frac{e}{\pi} \sin\frac{\pi z}{e} \tag{4}$$

It must be noticed that the classical homogeneous sinus model [13] can be recovered from Eq. (3) assuming $u_\alpha^1 = -u_{3,\alpha}^0$, and neglecting the unknown functions u_3^α.

The choice of the sinus function can be justified from the three-dimensional point of view, using the work [21]. As it can be seen in [22], a sinus term appears in the solution of the shear equation (see Eq. (7) in [22]). Therefore, the kinematics proposed can be seen as an approximation of the exact three-dimensional solution. Furthermore, the sinus function has an infinite radius of convergence and its Taylor expansion includes not only the third order terms but all the odd terms.

2.2.2. The Murakami's Zig–Zag terms

In order to evaluate the influence of Zig–Zag terms [7] in a high-order ESL model, the following displacement per layer (k) are added to Eq. (3):

$$\begin{cases} U_1^{(k)}(x_\alpha, z) = Z^{(k)}(z)\, u_1^z(x_\alpha) \\ U_2^{(k)}(x_\alpha, z) = Z^{(k)}(z)\, u_2^z(x_\alpha) \\ U_3^{(k)}(x_\alpha, z) = Z^{(k)}(z)\, u_3^z(x_\alpha) \end{cases} \tag{5}$$

with

$$Z^{(k)}(z) = (-1)^k\, \zeta_k(z) \quad \text{and} \quad \zeta_k(z) = \frac{2}{e_k}\left(z - \frac{1}{2}(z_k + z_{k+1})\right) \tag{6}$$