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Vibration analysis of laminated soft core sandwich plates with piezoelectric sensors and actuators

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ABSTRACT

In this paper we present a finite element model for the dynamic analysis of sandwich laminated plates with a soft core and composite laminated face layers, as well as piezoelectric sensor and actuator layers. The model is formulated using a mixed *layerwise* approach, by considering a higher order shear deformation theory (HSDT) to represent the displacement field of the compressible core and a first order shear deformation theory (FSDT) for the displacement field of the adjacent laminated face layers and exterior piezoelectric layers. Control laws are implemented and the model is validated for free and forced vibrations with results from the literature and the effect of the core transverse compressibility is assessed on modal damping and frequency response.

1. Introduction

From the early 1990s active constrained layer damping became an important subject of research [1,2]. These hybrid treatments combine the high capacity of passive viscoelastic materials to dissipate vibrational energy at high frequencies with the active capacity of piezoelectric materials at low frequencies. Therefore, in the same damping treatment, a broader control band is achieved [3,4]. The finite element method has been the main tool of solution for this class of problems, although analytical results have also been obtained for active control of beams and plates [5–7]. An extensive review on developments in active and passive constrained layer damping can be found in [8].

Sandwich plates with viscoelastic core are very effective in reducing and controlling vibration response of lightweight and flexible structures, where the soft core is strongly deformed in shear, due to the adjacent stiff layers. Hence, due to this high shear developed inside the core, equivalent single layer plate theories, even those based on higher order deformations, are not adequate to describe the behaviour of these sandwiches, also due to the high deformation discontinuities that arise at the interfaces between the viscoelastic core material and the surrounding elastic constraining layers. The usual approach to analyse the dynamic response of sandwich plates uses a layered scheme of plate and brick elements with nodal linkage. This approach leads to a time consuming spatial modelling task. To overcome these difficulties, the *layerwise* theory has been considered for constrained viscoelastic treatments, and more recently, Araújo et al. [9,10] and Moita et al. [11,12], among others, presented *layerwise* formulations for active sandwich plates with viscoelastic core and piezoelectric sensors and actuators. In these models, the effect of core compressibility is often overlooked. Hence in this paper a generalisation of the element developed by Araújo et al. [9] is presented for active and passive damping of soft core sandwich plates, where the transverse compressibility of the core is included [13,14]. The viscoelastic core layer is modelled according to a higher order shear deformation theory, adjacent elastic and piezoelectric layers are modelled using the first order shear deformation theory, and all materials are considered to be orthotropic, with elastic layers being formulated as laminated composite plies. Passive damping is dealt with using the complex modulus approach, allowing for frequency dependent viscoelastic materials. Feedback control laws are implemented for co-located control and the dynamic response of the finite element model is validated using reference solutions from the literature.

2. Sandwich plate model

The development of a *layerwise* finite element model is presented here, to analyse sandwich laminated plates with a viscoelastic (v) core, composite laminated face layers (e_1, e_2) and piezoelectric sensor (s) and actuator (a) layers, as shown in Fig. 1.

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The basic assumptions in the development of the sandwich plate model are:

1. The origin of the z axis is the medium plane of the core layer;
2. No slip occurs at the interfaces between layers;
3. The displacement is C^0 along the interfaces;
4. Elastic and piezoelectric layers are modelled with first order shear deformation theory (FSDT) and viscoelastic core with a higher order shear deformation theory (HSDT);
5. All materials are linear, homogeneous and orthotropic and the elastic layers (e_1) and (e_2) are made of laminated composite materials;
6. For the viscoelastic core, material properties are complex and frequency dependent;
7. Upper and lower layers play the roles of sensor and actuator, respectively, and are connected via feedback control laws, considering co-located control.

The FSDT displacement field of the face layers may be written in the general form:

$$\begin{aligned} u^i(x,y,z,t) &= u_0^i(x,y,t) + (z-z_i)\theta_x^i(x,y,t) \\ v^i(x,y,z,t) &= v_0^i(x,y,t) + (z-z_i)\theta_y^i(x,y,t) \\ w^i(x,y,z,t) &= w_0^i(x,y,t) \end{aligned} \tag{1}$$

where u_0^i and v_0^i are the in-plane displacements of the mid-plane of the layer, θ_x^i and θ_y^i are rotations of normals to the mid-plane about the y axis (anticlockwise) and x axis (clockwise), respectively, w_0^i is the transverse displacement of the layer, z_i is the z coordinate of the mid-plane of each layer, with reference to the core layer mid-plane ($z=0$), and $i = s, e_1, e_2, a$ is the layer index.

For the viscoelastic core layer, the HSDT displacement field is written as a series expansion of the displacements in the thickness coordinate:

$$\begin{aligned} u^v(x,y,z,t) &= u_0^v(x,y,t) + z\theta_x^v(x,y,t) + z^2 u_0^{*v}(x,y,t) + z^3\theta_x^{*v}(x,y,t) \\ v^v(x,y,z,t) &= v_0^v(x,y,t) + z\theta_y^v(x,y,t) + z^2 v_0^{*v}(x,y,t) + z^3\theta_y^{*v}(x,y,t) \\ w^v(x,y,z,t) &= w_0^v(x,y,t) + z\theta_z^v(x,y,t) + z^2 w_0^{*v}(x,y,t) \end{aligned} \tag{2}$$

where u_0^v and v_0^v are the in-plane displacements of the mid-plane of the core, θ_x^v and θ_y^v are rotations of normals to the mid-plane of the core about the y axis (anticlockwise) and x axis (clockwise), respectively, w_0^v is the transverse displacement of the core mid-plane. The functions $u_0^{*v}, v_0^{*v}, w_0^{*v}, \theta_x^{*v}, \theta_y^{*v}$ and θ_z^v are higher order terms in the series expansion, defined also in the mid-plane of the core layer.

The displacement continuity at the layer interfaces can be written as:

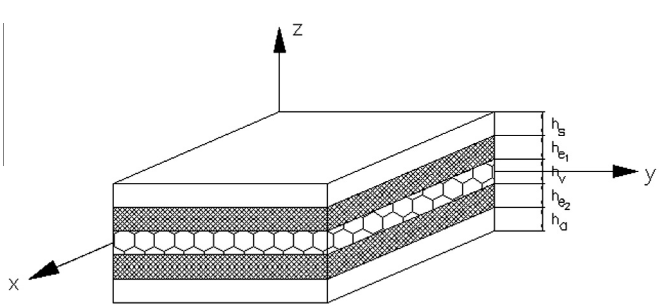

Fig. 1. Sandwich plate.

$$\begin{aligned} u^s\left(x,y,\frac{h_v}{2}+h_{e_1},t\right) &= u^{e_1}\left(x,y,\frac{h_v}{2}+h_{e_1},t\right) \\ v^s\left(x,y,\frac{h_v}{2}+h_{e_1},t\right) &= v^{e_1}\left(x,y,\frac{h_v}{2}+h_{e_1},t\right) \\ u^v\left(x,y,\frac{h_v}{2},t\right) &= u^{e_1}\left(x,y,\frac{h_v}{2},t\right) \\ v^v\left(x,y,\frac{h_v}{2},t\right) &= v^{e_1}\left(x,y,\frac{h_v}{2},t\right) \\ w^v\left(x,y,\frac{h_v}{2},t\right) &= w_0^{e_1} \\ u^v\left(x,y,-\frac{h_v}{2},t\right) &= u^{e_2}\left(x,y,-\frac{h_v}{2},t\right) \\ v^v\left(x,y,-\frac{h_v}{2},t\right) &= v^{e_2}\left(x,y,-\frac{h_v}{2},t\right) \\ u^a\left(x,y,-\frac{h_v}{2}-h_{e_2},t\right) &= u^{e_2}\left(x,y,-\frac{h_v}{2}-h_{e_2},t\right) \\ v^a\left(x,y,-\frac{h_v}{2}-h_{e_2},t\right) &= v^{e_2}\left(x,y,-\frac{h_v}{2}-h_{e_2},t\right) \\ w^v\left(x,y,-\frac{h_v}{2},t\right) &= w_0^{e_2} \end{aligned} \tag{3}$$

where the coordinates of layer mid-planes are:

$$\begin{aligned} z_s &= \frac{h_v}{2} + h_{e_1} + \frac{h_s}{2} \\ z_{e_1} &= \frac{h_v}{2} + \frac{h_{e_1}}{2} \\ z_v &= 0 \\ z_{e_2} &= -\frac{h_v}{2} - \frac{h_{e_2}}{2} \\ z_a &= -\frac{h_v}{2} - h_{e_2} - \frac{h_a}{2} \end{aligned} \tag{4}$$

Applying the continuity conditions, one obtains:

$$\begin{aligned} \theta_x^s &= \frac{2}{h_s}\left(u_0^s - 2u_0^{e_1} + u_0^v + \frac{h_v}{2}\theta_x^v + \frac{h_v^2}{4}u_0^{*v} + \frac{h_v^3}{8}\theta_x^{*v}\right) \\ \theta_y^s &= \frac{2}{h_s}\left(v_0^s - 2v_0^{e_1} + v_0^v + \frac{h_v}{2}\theta_y^v + \frac{h_v^2}{4}v_0^{*v} + \frac{h_v^3}{8}\theta_y^{*v}\right) \\ \theta_x^{e_1} &= \frac{2}{h_{e_1}}\left(u_0^{e_1} - u_0^v - \frac{h_v}{2}\theta_x^v - \frac{h_v^2}{4}u_0^{*v} - \frac{h_v^3}{8}\theta_x^{*v}\right) \\ \theta_y^{e_1} &= \frac{2}{h_{e_1}}\left(v_0^{e_1} - v_0^v - \frac{h_v}{2}\theta_y^v - \frac{h_v^2}{4}v_0^{*v} - \frac{h_v^3}{8}\theta_y^{*v}\right) \\ \theta_x^{e_2} &= \frac{2}{h_{e_2}}\left(-u_0^{e_2} + u_0^v - \frac{h_v}{2}\theta_x^v + \frac{h_v^2}{4}u_0^{*v} - \frac{h_v^3}{8}\theta_x^{*v}\right) \\ \theta_y^{e_2} &= \frac{2}{h_{e_2}}\left(-v_0^{e_2} + v_0^v - \frac{h_v}{2}\theta_y^v + \frac{h_v^2}{4}v_0^{*v} - \frac{h_v^3}{8}\theta_y^{*v}\right) \\ \theta_x^a &= \frac{2}{h_a}\left(-u_0^a + 2u_0^{e_2} - u_0^v + \frac{h_v}{2}\theta_x^v - \frac{h_v^2}{4}u_0^{*v} + \frac{h_v^3}{8}\theta_x^{*v}\right) \\ \theta_y^a &= \frac{2}{h_a}\left(-v_0^a + 2v_0^{e_2} - v_0^v + \frac{h_v}{2}\theta_y^v - \frac{h_v^2}{4}v_0^{*v} + \frac{h_v^3}{8}\theta_y^{*v}\right) \\ \theta_z^v &= \frac{w_0^{e_1} - w_0^{e_2}}{h_v} \\ w_0^{*v} &= \frac{4}{h_v^2}\left(\frac{w_0^{e_1} + w_0^{e_2}}{2} - w_0^v\right) \end{aligned} \tag{5}$$

These relations allow us to retain the translational degrees of freedom of the elastic and piezoelectric face layers, while eliminating the corresponding rotational ones. At the same time, the higher