Composite Structures 147 (2016) 1–15

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Review

A review of semi-analytical numerical methods for laminated composite and multilayered functionally graded elastic/piezoelectric plates and shells

Chih-Ping Wu *, Yan-Cheng Liu

Department of Civil Engineering, National Cheng Kung University, Taiwan, ROC

A R T I C L E I N F O

Article history:
Received 4 December 2015
Accepted 16 March 2016
Available online 17 March 2016

Keywords:
Review
Semi-analytical methods
Quasi-3D solutions
FGM
Plates
Shells

A B S T R A C T

The paper is to present an overview of various semi-analytical numerical methods for quasi-three-dimensional (3D) analyses of laminated composite and multilayered (or sandwiched) functionally graded elastic/piezoelectric materials (FGEMs/FGPMs) plates and shells with combinations of simply-supported, free and clamped edge conditions. This review introduces the development of various semi-analytical numerical methods incorporating 3D analytical approaches (i.e., the state space and asymptotic ones) with numerical techniques (i.e., the differential quadrature, meshless reproducing kernel and finite element ones), and their applications to the analyses of plates and shells made of advanced materials, such as the fiber-reinforced composite materials, FGEMs and FGPMs, and carbon nanotube-reinforced composite materials. Two micromechanical schemes (i.e., the rule of mixtures and Mori–Tanaka scheme) used to estimate the effective material properties of functionally graded structures are presented. The strong and weak formulations of the 3D piezoelectricity theory and their corresponding possible edge conditions for circular hollow cylinders are presented for the illustration purposes. A comparative study of the results obtained by using assorted semi-analytical numerical methods is undertaken.

Contents

* Corresponding author.
E-mail address: cpwu@mail.ncku.edu.tw (C.-P. Wu).

http://dx.doi.org/10.1016/j.compstruct.2016.03.031
0263-8223/

1. Introduction

A number of studies have carried out three-dimensional (3D) analyses of the mechanical/piezoelectric responses of simply-supported, laminated composite plates/shells and multilayered functionally graded material (FGM) ones with various reinforcements, such as micro-scaled continuous graphite, grass and boron fibers, and nano-scaled discrete carbon nanotubes and graphene sheets. Among these, however, relatively few papers consider the 3D analyses of these structures with various boundary conditions compared to those that examine structures with fully simply-supported edges. Wu et al. [1] classified the 3D exact approaches for simply-supported structures available in the literature into four different categories, namely, the state space [2–5], series expansion [6–9], Pagano [10–13], and perturbation methods [14–17]. Moreover, the related two-dimensional (2D) advanced and refined theories have been surveyed in a number of review articles by Noor and Burton [18–20], Noor et al. [21], Carrera [22–25], Liew et al. [26], Swaminathan et al. [27], Thai and Kim [28], Sayyad and Ghugal [29], and Jha et al. [30].

Some pure 3D numerical methods have been presented for the stress, deformation and free vibration analyses of the aforementioned structures with assorted boundary conditions, such as the 3D differential quadrature (DQ) [31–34], 3D finite element (FE) [35–43] and 3D meshless methods [44–46], even though these methods are very time consuming. Based on the strong formulations of the 3D elasticity and piezoelectricity theories, the DQ techniques [47–51] are used to interpolate each field variable in the spatial coordinates, such that the 3D basic equations and the corresponding boundary conditions can be transformed into a set of simultaneously algebraic equations in terms of the nodal field variables. The 3D DQ solutions can be thus obtained by solving the resultant equations and using the weighted least squares, Gaussian elimination, or LU decomposition methods. As for the 3D FE and meshless methods, the sets of shape functions are constructed by using Lagrange (or Hermite) polynomials and meshless methods (i.e., reproducing kernel (RK) [52–54], moving least squares [55–57] and radial basis functions [58–60] approaches), respectively, and these are used to interpolate (or approximate) the spatial variations of each field variable. By substituting these kinematic models in the weak formulations of 3D elasticity/ piezoelectricity theories, a set of simultaneous algebraic equations can be obtained by performing the numerical integration, and then the corresponding quasi-3D FE and meshless solutions will be obtained.

Due to the fact that the pure 3D analytical approaches are mathematically complicated, and that the solution processes use with pure 3D numerical methods are very time consuming, a class of compromise approaches, so-called 3D semi-analytical numerical methods, has thus been developed for the quasi-3D analyses of the structures with various boundary conditions. In these, the above-mentioned analytical methods are incorporated with these numerical methods to reduce the mathematical complexity occurring in pure 3D analytical approaches, and reduce the time needed with pure 3D numerical methods. In this review paper we will focus on the development of semi-analytical numerical methods incorporating the state space and asymptotic approaches with the DQ, RK and FE methods and their applications to the 3D static, free vibration and buckling analyses of laminated composite and multilayered (or sandwiched) functionally graded elastic/piezoelectric material (FGEM/FGPM) plates and shells, such as the finite layer, meshless differential RK (DRK) collocation, element-free Galerkin (EFG), sampling surfaces (SaS), finite prism, state space DQ, state space FE, state space DRK, asymptotic DQ, asymptotic FE and asymptotic DRK methods.

For illustration purposes, the strong and weak formulations of the 3D piezoelectric theory in the cylindrical shell coordinates and the appropriate possible boundary conditions are derived and presented by using Reissner's mixed variational theorem (RMVT) [61,62]. A comparative study of the results obtained by using different semi-analytical numerical methods is carried out, in which the accuracy and convergence of these methods are examined by comparing their solutions with the exact 3D and accurate 2D ones available in the literature.

2. Material properties

Without loss of the generality, the material properties in the formulations derived later in the work can be considered as heterogeneously varying through the thickness coordinate. The structural behavior of single-layered homogeneous, laminated composite, multilayered (or sandwiched) FGEM/FGPM and functionally graded (FG) carbon nanotube-reinforced composite (CNTRC) plates and shells can be studied by assuming appropriate material-property variations through the thickness coordinate of these. The material properties of the above-mentioned plates and shells are classified and given, as follows:

2.1. Single-layered homogeneous plates and shells

For a single-layered homogeneous plate or shell, the material properties, $m_{ij}(\zeta)$, are assumed as homogeneous, independent of the thickness coordinate, and are given by

$$m_{ij}(\zeta) = \text{constants}, \tag{1}$$

in which ζ denotes the global thickness coordinate located at the mid-surface of a cylindrical shell or a plate.

2.2. Multilayered homogeneous plates and shells

For a multilayered homogeneous plate or shell, the material properties are assumed to be layerwise Heaviside functions and are given by

$$m_{ij}(\zeta) = \sum_{m=1}^{N_l} m_{ij}^{(m)} [H(\zeta - \zeta_{m-1}) - H(\zeta - \zeta_m)], \tag{2}$$