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Element-free geometrically nonlinear analysis of quadrilateral functionally graded material plates with internal column supports

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A B S T R A C T

A nonlinear deflection analysis is carried out for internal column supported functionally graded material (FGM) arbitrarily straight-sided quadrilateral plates under a uniformly distributed loading. In order to achieve this aim, the formulation of a discrete nonlinear governing equation for large deformation is undertaken based on the element-free IMLS-Ritz method. The first-order shear deformation theory and von Kármán assumption are adopted in the formulation. In this study, the computation involves a geometric transformation from the physical quadrilateral domain to a square computational domain. Several example problems are selected to illustrate the effect of internal column support distributions on the maximum deflection and deformed shape of the FGM plates. In addition, parametric studies on the effect of volume fraction ratio, geometry and length-to-thickness ratio on the large deflection behavior of FGM plates are examined.

1. Introduction

About three decades ago, a group of materials scientists [1,2] proposed the concept of functionally graded materials (FGMs). It is known that FGM is a type of heterogeneous composite material whose material properties vary gradually from one surface to another in a predetermined engineered pattern. It is fabricated by mixing two discrete phases of materials – typically a mixture of ceramics and metals. This tailor-made composite can withstand a high-temperature environment due to its enhanced thermal resistance, provided by the ceramic constituents. Meanwhile, the metal constituents provide enhanced mechanical performance by reducing the possibility of catastrophic fracture. Because of its superior mechanical properties, FGM has been made in the form of beam, plate and shell, and has a vast catalogue of applications to structural elements in special nuclear components and spacecraft structural members.

Extensive studies have been conducted on the linear mechanical analysis of FGM structural elements. Filippi et al. [3] employed the finite element method to study the static analysis of FGM beams using various theories. Neves et al. [4] proposed a quasi-3D sinusoidal shear deformation theory for static and vibration analyses of FGM plates. In a later study, Neves et al. [5] used a quasi-3D higher-order shear deformation theory for static, vibration and buckling analyses of sandwich FGM plates using a meshless method. Sofiyev [6] considered the vibration and stability of shear deformable FGM truncated conical shells subjected to an axial load. Zhao et al. [7–9] employed the element-free kp-Ritz method for static, free vibration and buckling analyses of FGM plates and shells.

Asemi et al. [10] developed a full compatible three-dimensional elasticity element for the buckling analysis of FGM rectangular plates subjected to various combinations of biaxial, normal and shear loads. Mansouri and Shariyat [11] examined the biaxial thermo-mechanical buckling behavior of orthotropic auxetic FGM plates with temperature and moisture dependent material properties on elastic foundations. Zhang et al. [12] used a local Kriging meshless method to study the thermal buckling of functionally graded plates. Malekzadeh [13] employed the differential quadrature method to study the three-dimensional thermal buckling of functionally graded arbitrary straight-sided quadrilateral plates. Malekzadeh and Alibeygi Beni [14] studied the free vibration of functionally graded arbitrary straight-sided quadrilateral plates in a thermal environment.

Numerous studies have also been conducted on the nonlinear mechanical analysis of FGM structural elements. Praveen and

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Reddy [15] presented the nonlinear transient thermoelastic solution for functionally graded ceramic metal plates. Zhao and Liew [16] considered the geometrically nonlinear analysis of functionally graded plates using the element-free kp-Ritz method. Shen and Wang [17] studied the nonlinear vibration of shear deformable FGM cylindrical panels resting on elastic foundations in thermal environments. Tung [18] reported the thermal and thermomechanical postbuckling of FGM sandwich plates resting on elastic foundations with tangential edge constraints and temperature dependent properties.

Wu et al. [19] employed finite double Chebyshew polynomials to obtain an analytical postbuckling solution for rectangular FGM plate subjected to thermal and mechanical loads. Lee et al. [20] used the element-free kp-Ritz method to study the postbuckling behavior of rectangular FGM plates subjected to edge compression. Lal et al. [21] examined the second-order statistical postbuckling behavior of rectangular FGM plates under thermal and mechanical loads. Duc and Cong [22] derived an analytical postbuckling solution for thick rectangular FGM plates resting on elastic foundations, in which the plates were subjected to thermos-mechanical loads in thermal environments.

Zhang and Zhou [23] studied the mechanical and thermal postbuckling behaviors of rectangular FGM plates resting on nonlinear elastic foundations. Zhu et al. [24] developed a local Petrov–Galerkin approach with moving Kriging interpolation to examine the geometrically nonlinear thermomechanical behavior of moderately thick functionally graded plates. Asemi et al. [25] studied the postbuckling behavior of FGM annular sector plates based on three-dimensional elasticity graded finite elements.

From the known literature [3–29], it is aware that no existing work has been conducted on the geometrically nonlinear large deformation of FGM plates of arbitrary straight-sided quadrilateral shape with internal supports. However, we recognize the fact that many nuclear and spacecraft applications involving FGM plates utilize internal supports to provide strength to the structures. Therefore, the primary aim of the present study is to examine the effects of internal supports on the large deflection of arbitrarily straight-sided quadrilateral shaped FGM plates.

In this study, the mathematical formulation of the discretized governing equations are derived using the element-free IMLS-Ritz method [30–34] based on the first-order shear deformation theory and von Kármán assumption. Nonlinear solutions to the discretized governing equations are furnished by the hybrid arc-length iterative procedure with the modified Newton-Raphson method. Parametric studies on the effects of various parameters, such as plate thickness-to-width ratio, geometry and volume fraction ratio, on the large deformation behavior of FGM plates under various boundary conditions are considered. In particular, the effects of the distribution patterns of the internal column support on the maximum deflection and deformed shape of the FGM are examined.

2. Modeling of FGM plates with column supports

Consider a quadrilateral FGM plate of length a, widths b and c, total thickness t, and side angles α and β. A Cartesian coordinate system is established on the middle plane of the arbitrary straight-sided quadrilateral FGM plate, as shown in Fig. 1. The material properties, including Young's modulus E, Poisson's ratio v, thermal conductivity κ and thermal expansion coefficient α, are assumed to vary along the thickness direction according to the volume fractions of the constituents. It is worth noting that Poisson's ratio v depends weakly on temperature change and position, and so it is taken to be a constant in general calculations. According to the model of FGM rectangular plates [16] based on

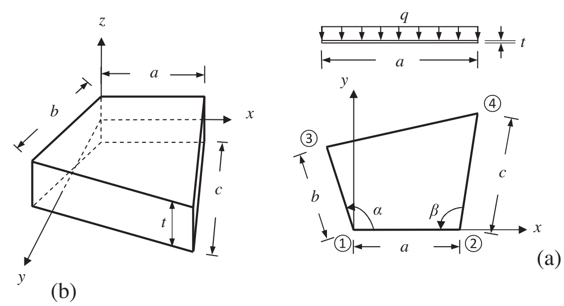

Fig. 1. Configuration of a quadrilateral FGM plate under a uniformly distributed loading in (a) x, y, z coordinates and (b) x, y coordinates.

the first-order shear deformation plate theory (FSDT) [35], the displacements, strains and stresses have the same form as found in previous studies. For the sake of simplicity, the deducing process of the formulae is omitted and the total potential energy functional of the plate can be expressed as

$$\Pi = \frac{1}{2}\int_{\Omega} \varepsilon^T \mathbf{S} \varepsilon d\Omega - \int_{\Omega} \mathbf{u}^T \bar{\mathbf{f}} d\Omega - \int_{\Gamma} \mathbf{u}^T \bar{\mathbf{t}} d\Gamma, \tag{1}$$

where

$$\mathbf{S} = \begin{bmatrix} \mathbf{A} & \bar{\mathbf{B}} & \mathbf{0} \\ \bar{\mathbf{B}} & \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_s \end{bmatrix}, \tag{2}$$

and $\bar{\mathbf{f}}$ represents the external load, $\bar{\mathbf{t}}$ is the prescribed traction on the natural boundary and $\mathbf{u}$ is the displacement vector.

The matrices $\mathbf{A}$, $\bar{\mathbf{B}}$, $\mathbf{D}$ and $\mathbf{A}_s$ in Eq. (2) are the in-plane, bending-stretching coupling, bending and shear stiffnesses, and their corresponding terms are given as

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-t/2}^{t/2} Q_{ij}(1, z, z^2) dz, \tag{3}$$

$$A_{ij}^s = K \int_{-t/2}^{t/2} Q_{ij} dz, \tag{4}$$

where A_{ij}, B_{ij} and D_{ij} are defined as i, j = 1, 2, 6 and A_{ij}^s is defined as i, j = 4, 5. For functionally graded materials, the transverse shear correction coefficient K is taken to be $K = 5/(6 - (\upsilon_1 V_1 + \upsilon_2 V_2))$ by Efraim and Eisenberger [36], where V_1 and V_2 represent the volume fraction of each material over the entire cross-section, and υ_1 and υ_2 are the Poisson ratios of the two materials.

The strain-displacement relations are expressed as

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \boldsymbol{\varepsilon}_0 \\ \boldsymbol{\kappa} \\ \boldsymbol{\gamma}_0 \end{Bmatrix}, \tag{5}$$

in which the strain components at a generic point are expressed as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \boldsymbol{\varepsilon}_0 + z\boldsymbol{\kappa}, \tag{6}$$

and

$$\boldsymbol{\varepsilon}_0 = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix}, \tag{7}$$