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A new analytical model for evaluating interlaminar stresses in the unfolding failure of composite laminates

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ABSTRACT

Curved laminates of composite materials are analytically investigated in this paper. Curved zones are a critical part of many kinds of composite beams, for example, L-shaped, C-shaped, Ω-shaped or T-shaped beams, which are susceptible to unfolding failure. A new stress calculation procedure is presented to determine the radial, circumferential and shear stresses in a curved laminate under normal and tangential surface loads and tension, out of plane shear and bending end loads, assuming a 2D stress state. The method assumes that the laminate is made up of a set of fictitious laminas. Making the number of fictitious laminas tend to infinity, analytical solutions are obtained. The nature of the method makes it extremely easy to apply to composite laminates. Firstly, the accuracy of the method is proved by comparing its results with those obtained from the literature for the problems concerning the pure bending of a homogeneous and a layered curved beams. Secondly, the results obtained using this analytic method with the problem of a layered curved beam subjected to surface loads (for which no previous closed-form analytical solution has been found in the literature) are compared with the results obtained using a finite element model, showing excellent agreement.

1. Introduction

The use of composite materials in the aerospace industry has significantly increased over the last years. Composite laminates have been introduced into the design and manufacturing of aircraft components with high responsibility. The main advantage of composite laminates is their high specific strengths associated with their in-plane stresses. However, the massive use of composites in aircraft structures involves their usage in more complex geometries. Therefore, a better knowledge of the behaviour of composite laminates under complicated stress fields is required.

Composite laminates are well designed for bearing high intralaminar stresses, being very efficient in these kind of situations. However, their interlaminar strengths are comparatively much lower than the intralaminar ones. In high curvature zones of composite material components, interlaminar stresses develop which, added to the low strengths associated with the intralaminar plane, give rise to the appearance of undesirable failure modes. This is the case of the unfolding failure, which consists of a delamination observed when the curved component is loaded under an opening bending moment. Typical components prone to this kind of failure (see [1,2]) are angle brackets, T-shaped beams, joggles, Ω-shaped beams, corrugated laminates, etc.

The Classical Laminate Theory (see [3, chap. 4]) does not consider interlaminar stresses, either because typical laminate designs only consider intralaminar stresses or because the interlaminar stresses are too small when compared with the intralaminar ones. This classical method has also been applied to curved laminates [4], but is again not able to calculate the interlaminar stresses. However, when the laminate is curved with $t \sim R$ (where t is the thickness of the laminate and R the mean radius), interlaminar stresses may reach significant values and become the main failure cause in these kinds of composite laminates.

Typically, these interlaminar stresses are calculated using Lekhnitskii's equations (see [5, chap. 3]), which yield very accurate results in many cases. These equations are widely used in the literature [6–10], but their main limitation is that they are valid only for anisotropic homogeneous materials and composite materials are not homogeneous at the laminate level (inhomogeneities associated with the presence of fibre and matrix inside each ply are not considered in this paper). Thus, the less homogeneous the laminate in terms of the stacking sequence, the more inaccurate the results when applying Lekhnitskii's equations.

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Moreover, Lekhnitskii's equations are obtained for constant curvature 2D curved beams by assuming either that stresses are not dependent on the angular co-ordinate (valid when the beam is under a constant bending moment) or that stresses have a sinusoidal dependence on the angular co-ordinate (valid when the beam is under end forces and moments). Due to these hypotheses, Lekhnitskii's equations are not valid when distributed non-auto-equilibrated loads are applied, because these cause stresses to have a non-sinusoidal dependence on the angular co-ordinate.

Using the same hypotheses, Ko and Jackson [11] developed the equations for the evaluation of stresses in a curved composite laminate under end loads. To decrease the complexity of applying the equations developed by Ko and Jackson, Kedward et al. [1] presented a simplified and approximated equation for the maximum of the radial stress due to the bending moment, and Cui et al. [12] the corresponding approximation for the maximum shear stress due to the shear force (approximating it using the maximum in a flat laminate).

Using the aforementioned equations, the design procedure for a composite component prone to unfolding failure is typically carried out by: (1) calculating the forces and moments in the design loading state, (2) calculating the stresses by using the previously mentioned solutions and (3) applying a failure criterion, typically given as a function of the stresses and their allowable values.

The first unfolding failure criteria were based on mean stresses in the thickness (see [13]). Later, more advanced criteria are: firstly the one presented by Kim and Soni [14], which consists of a quadratic criterion concerning the shear and normal interlaminar stresses, and secondly the one presented by Most et al. [15], which also includes the shear and normal interlaminar stresses, but uses different quadratic criteria on the tensile and compressive sides. Other authors also include the stress along the fibre direction in the delamination criterion [16].

The unfolding failure criteria require the strengths of the material in the interlaminar direction to be determined. The interlaminar tensile strength (ILTS) is typically calculated by the four-point bending test [17,18]. It can also be calculated using a tensile test [19], but the ILTS calculated by this and other methods based on straight beams is usually very high when compared with the ILTS obtained for the curved beam [20], mainly due to the size effect [21]. The interlaminar shear strength (ILSS) is typically calculated using a three-point bending test [22].

The design procedure described above produces very conservative results in many cases (since the stress solutions employed do not take into account the edge-effects, that is, the changes in the interlaminar stresses appearing at the beam ends or in the vicinity of the connection between beams with different curvatures). Therefore, finite elements calculation is usually employed for a more precise stress calculation. As an example, see Most et al. [15], who showed that actual analytical procedures are very conservative at the zones in which the curvature of the beam changes (typical in sections such as those represented in Fig. 1).

Due to the conservative results obtained and to the difficulties found in extending the analytical model to more complex geometries (double-curvature, variable curvature, distributed loads, corrugated, etc.), several authors have been working on the development of specific shell elements for the finite elements method [23–25] and other numerical calculation techniques such as the differential quadrature method [26,27] and power series expansion of displacements and stresses [28]. However, the finite elements method and other numerical techniques are not efficient in the design phase of composite structures, since high calculation and modelization times are required for the optimization of the geometry and the stacking sequence. Therefore, analytical models for quick stress calculations are required to improve efficiency in the design phase.

Typical analytical curved beam calculation methods consider the hypothesis for thin beams ($t \ll R$), which requires the calculation of the stiffness constants similar to the calculation in the classic flat laminate theory, see Lin and Lin [29]. These methods offer a preliminary estimation of the interlaminar stresses. However, many curved beams have a R/t ratio between 1 and 5 and, therefore, the hypothesis for thin beams is not always valid. In these cases several authors have considered the effect of the curvature on the stiffness constants and the strain–displacement relations, for example Qatu [30], Kress et al. [31] and Guedes and Sa [32], who all obtain logarithmic expressions for the stiffness constants A, B and D of the laminate due to the curvature effect. These logarithmic expressions imply a coupling between in-plane and bending effects ($B \neq 0$) even in the presence of symmetrical stacking sequences. These models are approximations and simplifications of the exact model developed by Ko and Jackson [11].

The analytical solutions were compared with experimental results in several geometries and loading states. For example, Cui and Ruiz [33] and Wisnom et al. [34] tested C-specimens under combined axial, shear and bending loads, Cui et al. [12] tested curved beams under pure bending, and McRobbie et al. [35] tested open-rings of composite laminates and sandwiches under bending due to end loads.

Going back to the solution given by Ko and Jackson [11], this presents two main limitations. Firstly, it only takes into account end loads, and its extension to other cases of interest such as pressure loads and shear distributed loads is not straightforward since the required stress functions have not been obtained yet. Secondly, it is based on regularized solutions and, therefore, does not take into account the edge-effects showed by Most et al. [15].

In this paper, an analytical model is proposed for the development of a new calculation methodology for stresses in curved laminated beams, taking into account the heterogeneity of a composite laminate across the thickness. For the analysis of a curved laminate subjected to end loads, this model has the advantage of being easier to implement than the analytical solution given by Ko and Jackson [11], while at the same time maintaining greater accuracy for moderately thick curved composite beams. In addition, it has the capability of overcoming the aforementioned limitations of the analytical solution given by Ko and Jackson [11], since it can be easily extended to calculate stresses due to external pressure and shear surface loads (as is shown in the present paper), in computational times smaller than other analytical models such as the one presented by Matsunaga [28]. Although not shown in this paper, due to lack of space, it also offers the possibility of being expanded to calculate edge-effects and even to consider 3D stress states.

Firstly, the fundamentals of the model are developed for the case of a curved beam made of a homogeneous anisotropic material under end loads (for both thin and thick laminates). If external pressure and shear surface loads are null and sections sufficiently far from the beam ends are analysed, the solution presented by Lekhnitskii et al. [5] can be considered exact, and thus is employed as a benchmark to show the superior accuracy of the method developed for distinct thicknesses.

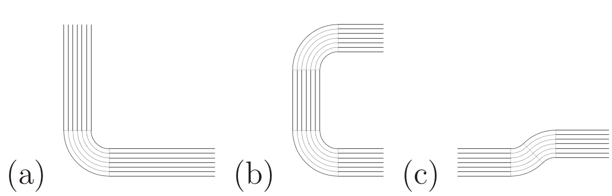

Fig. 1. Typical composite sections made up of straight and curved parts: (a) L-shaped beam, (b) C-shaped beam, (c) joggle.