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Increase of contact radius due to deflection in low velocity impact of composite laminates and prediction of delamination threshold load

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A B S T R A C T

An analytical method is proposed to calculate the increase of contact radius due to the deflection of the laminate subjected to low velocity impact with respect to that calculated with Hertz contact law. The method is validated with finite element method (FEM). The model of delamination threshold load (DTL) is improved based on the corrected contact radius. The low velocity impacts were conducted on a composite wing box, and the DTLs of skins were measured under the impacts of three indenters variant in diameter. The linear model was fist employed to prediction the DTLs, the maximum error of which reaches 27%. The increase of contact radius was calculated for each impact, which is ranging from 41% to 44%. The DTL were then recalculated based on the revised contact radius, and the prediction errors are lower than 6.5%. The presented method is less time consuming in comparison to the FEM and is applicable to predict the DTLs of a laminate family when the interlaminar shear strength is given.

1. Introduction

Composite materials such as carbon fiber reinforced polymer (CFRP) laminate are now widely used in engineering structures with high safety requirements. In the operation and maintenance, composite structures are exposed to the impact due to runway debris, hailstone and dropping tool, etc. The impact damage resistance is an important property of composite materials. The delamination caused by low velocity impact is a primary damage pattern of the laminate, for it is generally invisible or barely visible for the inspector but reduces the flexural rigidity which may lead to the premature buckling failure [1,2]. So that the delamination threshold load (DTL) is one of the primary metrics for the damage resistance [3] and is required by the standard of impact test [4]. In the low velocity impact, the DTL is the value of the time-load or the displacement-load curves where a sudden load drop occurs as a result of the loss of flexural rigidity due to delamination growth.

Some damage criteria were proposed based on the stresses or strain usually obtained through numerical method such as finite element method (FEM). These delamination criteria are B-K law [5,6], Hashin failure criteria [7,8], and so on, which are generally employed by LS-DYNA3D [9], ABAQUS [10] and can be also embedded in the laminate model via the user-defined material subroutine.

The analytical DTL model was also focused on in decades. Davies and Robinson [11] first proposed an equation of critical stress in terms of the delamination size and the impact force based on the interlaminar shear strength (ILSS). The equation shows a continuous map of damage area and impact force, which cannot describe the threshold behavior of impact force. Thereafter the fracture theory was introduced into the DTL model [12,13]. The laminate is regarded as a quasi-isotropic material and delaminations are represented with a single mid-plane delamination. As the critical energy release rate for mode II shear fracture is expressed in terms of the transverse load and the flexural rigidity of plate, the load at which the delamination occurs is obtained. Further, Suemasu, and Majima [14,15] derived a more accurate equation for the multiple delamination propagation of circular axisymmetric plates. Olsson [16] proposed the effective flexural stiffness for the impact response model of orthotropic composite plates. Based on the fracture theory, Olsson et al. [17–19] focused their researches on the analytical models of delamination initiation and growth for the laminate under the quasi-static and dynamical impact.

The fracture mechanics based model does not concern the impactor shape, but which were proved to be influential on the response and the damage of the composite laminate [20–23]. The Hertz contact law is feasible in modeling the local stress and deformation of the contact between two isotropic elastic bodies [24]. The contact of two transversely isotropic body was firstly

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investigated with respect to composite materials [25]. The complex models on contact stress and deformation of the orthotropic plate were also setup [26,27]. The Hertz contact law also well presents the contact behavior of carbon epoxy laminates [28,29]. Though the contact area of the orthotropic plate is elliptical, the ratio of the ellipticity of contact area is not a strong function of the orthotropic material properties. It is shown that the axes ratio of elliptical area ranges from 0.93 to 1.07 when the ratio of in-plane modulus is below 11.5 [30,31].

The interlaminar damage are mainly caused by the shear stress. The shear stress varies through the thickness and can be obtained using the closed form approximation [32]. It is easy to understand that the transverse shear force on the boundary of contact area is equal to the exerted load from the beam subjected to the impact a cylindrical indenter [33]. Similarly, the resultant shear force around the perimeter of contact area is also equal to the impact force. Assuming the impact force is uniformly distributed through the thickness, the relationship between the impact force and the shear stress is setup based on the Hertz contact law [34]. The delamination occurs as the average shear stress exceeds the interlaminar shear strength. The distribution of shear stress is greatly simplified in this method, but the peak value is proportional to the average one. So that it is shown that the DTL follows the model expressed in terms of plate thickness and indenter radius [35].

The plate *wraps* the indenter with the surface around the contact area when it is deflected due to the transverse load, especially when the flexural rigidity of plate is low. The *wrapping effect* increases the contact area and then decreases the shear stress around the perimeter. The similar effect was investigated for beams [36]. For the plate, the effects of plate deflection on the relationship between contact force and indentation were investigated [37–39]. Hence, the DTL obtained with simple ILSS based model may be lower than the real one under the *wrapping effect*. The size of contact area under the influence of deflection should be calculated to obtain the more accurate average shear stress.

In the remainder of this paper, a method to calculate the enlargement of contact radius is first proposed. The method is then verified through calculating the contact radius of laminate and composite half space with FEM. The impact tests were carried out on the wing box with skins of various thickness. The ILSS is obtained from the DTL of a certain skin and then is employed to compute the DTL of other skins with the pristine ILSS based model and the improved method, respectively. Then the results of the two methods are discussed. The summary and concluding remarks are given in the last section.

2. Theory

2.1. Basic delamination threshold load model

The contact force, P, between two smooth body solid bodies of revolution is usually related to the indentation by the Hertz contact law [24]

$$P = K_h \alpha_0^{3/2} \tag{1}$$

where K_h is the Hertz contact stiffness, α_0 is the indentation. For a spherical isotropic indenter and a transversely isotropic composite plate, n is

$$K_h = \frac{4E\sqrt{R}}{3} \tag{2}$$

where R is the radius of indenter, E is determined by Young's modulus and Poisson's ratio of the target and indenter

$$\frac{1}{E} = \frac{1-\nu_i^2}{E_i} + \frac{1-\nu_{12}\nu_{21}}{E_p} \tag{3}$$

where E_i and E_p are Young's modulus of the indenter of the impactor and composite plate, respectively; ν_i is Poisson ratio of the indenter; ν_{12} and ν_{21} are Poisson ratio of the composite laminate. Substituting Eq. (2) into Eq. (1) and rearranging give

$$\alpha_0 = \left(\frac{3P}{4E\sqrt{R}}\right)^{2/3} \tag{4}$$

The radius of contact area, a, is given as

$$a = \left(\frac{3PR}{4E}\right)^{\frac{1}{3}} \tag{5}$$

Assuming the shearing load P is uniformly distributed over the cylindrical surface extruded perpendicularly to the plate with the perimeter of contact area, thus the average shear stress, $\bar{\tau}_{rz}$, follows as

$$\bar{\tau}_{rz} = \frac{P}{2\pi ah} \tag{6}$$

where h is the thickness of plate. The delamination occurs as the shear stress exceeds the interlaminar shear strength S_{rz}, and the corresponding contact force is the delamination threshold load, i.e. P_{cr}. Substituting Eq. (5) into Eq. (6) and rearranging yields

$$P = \sqrt{\frac{6\bar{\tau}_{rz}^3\pi^3h^3R}{E}} \tag{7}$$

therefore P_{cr} can be written as

$$P_{cr} = \sqrt{\frac{6S_{rz}^3\pi^3h^3R}{E}} \tag{8}$$

It is reasonable to assume E and S_{rz} are constant for the laminates made from the same materials, thus the quantity in terms of them in Eq. (8) is a constant, i.e.

$$C = \sqrt{\frac{6S_{rz}^3\pi^3}{E}} \tag{9}$$

Hence P_{cr} is expressed in terms of C, thickness and the indenter radius

$$P_{cr} = C\sqrt{h^3R} \tag{10}$$

2.2. Delamination threshold load on considering the wrapping effect

Neglecting the *wrapping effect*, the nominal shear strength can be derived with Eq. (8) once the DTL of a laminate under the impact of an indenter is measured, then the DTL of other laminates under the impact of variant indenters can be calculated with Eq. (10). Whereas the global deflection of laminate may increase the contact area thus reduces the shear stress, hence the contact radius should be corrected in order to obtain the more accurate DTL.

The indentation and deflection of a laminated plate subjected to the impact of a rigid indenter is shown in the Fig. 1. The contact radius without the plate deflection is a, while it increases to a^* due to the wrapping of the deflected plate. The deflections of the plate at these two locations are w_a and w_{a^*}, respectively. The indentation is α_o at the center of contact area, and α_{a^*} at a^*. The relationship between them is given by

$$w_o + \alpha_o = w_{a^*} + \alpha_{a^*} + R - \sqrt{R^2 - a^{*2}} \tag{11}$$

Neglecting α_{a^*} and rearranging Eq. (11) yields

$$a^* = \sqrt{R^2 - (R - w_o + w_{a^*} - \alpha_o)^2} \tag{12}$$