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Review

The effect of approximation accuracy of the orientation distribution function on the elastic properties of short fibre reinforced composites

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ABSTRACT

In present paper the authors are focusing on the features of the orientation distribution function (ODF), used to give a probabilistic assessment to the orientation of fibres in fibre reinforced composites. The ODF can be approximated using experimental data and used to evaluate the anisotropic effect of fibres in considered directions. The hardened orthotropic elastic properties of steel fibre reinforced concrete (SFRC), evaluated based on the approximated ODF with different orders of accuracy are used as a case study. Input data for the approximation of the ODF are individual orientations of fibres measured by X-ray micro-tomography method. The study outcomes revealed that the orthotropic elastic properties of SFRC may not be estimated accurately enough by using only the second order terms in the reconstruction of the ODF. The accuracy of the stiffness matrix components is sensitive for the orientation distribution type and the approximation of the ODF, and this should be considered in all FE applications using the ODF approach. Besides, within the study the relation between the values of the scalar-order parameter *S*, evaluating how well the fibres are aligned with each other, and the approximation orders of the ODF is established.

Contents

1. Introduction

The characteristics or properties of many materials such as density or strength can be expressed by a single numeric value. The situation becomes complex when a material is composed of a mix of several different materials forming a composite. The advantage of mixing is the opportunity to improve the characteristics and properties of the original material-matrix-by including reinforcing elements of another more advanced material. Unfortunately, the latter results in a complex material micro-structure, which is difficult to model. For example, when incorporating short fibres into a matrix then one of the most relevant parameters determining the elastic properties of a composite is the orientation distribution of fibres. One aspect of the problem is to model fibre orientations and include them into the governing equations to consider the influence of fibres. In this paper we would like to concentrate on

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challenges related to the modelling of fibre orientations in the composites reinforced by short fibres. Recently, some constitutive models for short fibre reinforced composites have been already published in [9,14,17].

One way to model the orientation of short fibres is to employ the statistical orientation distribution function, (ODF), which is defined on a unite sphere [1,18,20]. The ODF is also successfully used for representing the orientation distribution of penny-shaped cracks [20], liquid crystals [19,22,24], oriented polymers [3,6], etc. demonstrating a multi-applicability of the ODF approach, which means that it is not restricted to any particular physical problem. This fact makes the ODF to be universal, which from one side can simplify the modelling part, but from the other side can cause physically meaningless results, such as negative outcomes appearing due to angular ranges having very low or close to zero probability density [9,24]. More often this situation appears when orientation of fibres is not randomly distributed, but shows tendencies to alignment, for example, as in concrete reinforced with short steel fibres, (SFRC) [8,9,11]. This composite is formed by mixing of fresh concrete with short steel fibres, which are added to the mass during the mixing stage. Steel fibres reinforce the concrete by changing its hardened state properties from brittle to more ductile resulting in improved tensile strength and crack resistance. The elastic properties of SFRC among others depend on the dominant orientation of fibres, which can be a result of many factors, of which the most significant ones are the chemical composition of cement clinker and the use of possible admixtures, which specifies the hydration process in general, the used aggregate size, casting technology and geometry of a structural element. In this paper we are using as a case study the hardened orthotropic elastic properties of SFRC, which are evaluated employing the ODF approach.

2. State of the art

The ODF, $f(\mathbf{n})$, is defined on a unit sphere. It belongs to the square-integrable functions, and it can be expanded into an infinite series of spherical harmonics. The l-order symmetric alignment tensors (ATs), $A_{\mu_0\dots\mu_l}$, are acting as expansion coefficients and the l-order symmetric irreducible tensors, $\overline{n_{\mu_0}\dots n_{\mu_l}}$, serve as complete orthonormal basis functions (main spherical functions) [7,18]. The l-order symmetric ATs are the traceless orientation tensors [1], which are calculated as the l-order outer products of a unit vector $\mathbf{n}$, representing space orientation of a rod-like particle, with itself, and then integrating the result with the ODF. The l-order symmetric irreducible tensors-basis functions-are also traceless, but those are calculated considering all possible directions of a rod-like particle on a unit sphere, i.e. $\mathbf{n} \in (\theta, \phi), \theta \in [0^\circ, 180^\circ]$, $\phi \in [0^\circ, 360^\circ]$. The ODF is even meaning that the odd order tensor terms are vanishing. The infinite series of the ATs in complete orthogonal basis can recover the ODF [1,13]. The ODF of fibres can also be approximated employing experimental data, as, for example, the measured fibre orientations [9,11,23]. In this case the infinite series of the ATs is replaced by a partial sum of finite terms.

The accuracy of a reconstructed ODF to represent the actual orientation distribution of fibres largely depends on the character of the arrangement of fibres in a matrix. If the fibres have a more isotropic distribution then presenting their orientation by an approximated ODF may require only the second order terms in the tensor series expansion, whereas in case of a more aligned distribution the tensor series should include higher order terms [24]. Interesting results were received by the authors in [12] when studying the influence of the lower and higher order terms in the approximation series of the ODF on the ability to represent the actual orientation distribution of fibres. The authors demonstrated with a planar tendency of fibre orientation distribution that the second order orientation tensor is not sensitive to variations in the in-plane angle ϕ and can stay invariant, while the fourth order orientation tensor is able to detect the changes within the ϕ angle, which results in different values for tensor components. An interesting fact was also that the discrepancy between the exact and the reconstructed ODF increases when the fibres are aligned in some preferred direction or plane. This highlights the importance of the higher order tensor terms in the approximation series of the ODF. Furthermore, mostly aligned distributions more often cause a negative part within the ODF [24]. This situation can be explained by orientation distribution of fibres, where some angular ranges have very low or close to zero probability density.

However, considering the features of the ODF presented and resulting from its study and application on the orientation of short fibres in polymer composites or on liquid crystals [1,13,19,24], it is hardly possible to recognise how the accuracy of approximation may affect if the ODF is employed to evaluate the elastic parameters of a material in its principle axes. For example, in case of SFRC the elastic properties are directly dependent on the effectiveness of the orientation of steel fibres in considered direction, and thus the approximation accuracy of the ODF should be of high importance [11,15]. A scalar-order parameter S, which is calculated based on the eigenvalues of the second order AT and which is a measure of the degree of alignment of rod-like particles, can be used to quantify a character of the orientation distribution [2,21]. The values of S range between $S \in [-0.5, 1.0]$ and, depending on a type of the orientation distribution, can have the following values in the limit cases: $S = 1.0$ transversely isotropic distribution, $S = 0$ isotropic distribution and $S = -0.5$ plane isotropic one. For the practical use of SFRC it is important to quantify the accuracy of the approximation of the ODF depending on the values of scalar-order parameter S.

Within the present study, SFRC orthotropic elastic characteristics are evaluated employing the ODF, which is based on the individual orientations of fibres measured by mCT method, and approximated using different orders of accuracy. SFRC samples containing individual fibres were extracted from the full size floor slabs, which were manufactured according to site specific casting [23]. Besides, the relation between the values of the scalar-order parameter S and the accuracy of the approximation of the ODF is established.

3. Different order reconstructions of the ODF from the measured fibre orientations in SFRC

In this study, the eigenvectors $\{\mathbf{d}^i\}_{i=1,2,3}$ of the second order AT, representing the dominant fibre orientations, were used to specify the orthotropic material symmetry axes of SFRC, and the ODF was used to evaluate the orthotropic elasticity of SFRC in the coordinate system defined by these symmetry axes [9]. To demonstrate the practicality of the approach described for material modelling, the ODF of fibres was approximated utilising the terms of the second order tensor, which were calculated employing the measured fibre orientations [8,9,11]. SFRC cylinder samples extracted from the full-size floor slabs were scanned using X-ray micro-computed tomography (mCT) method. Further, the scanning data were processed and analysed thus providing an accurate information on fibre orientations in the samples [8,23]. The position of each measured fibre was specified by spherical coordinates with the inclination angle θ and azimuth (in-plane) angle ϕ. The second order AT was calculated as a mean outer product of the measured orientation of an individual fibre with itself and removing the trace at the end. The sum expression of the second order AT for the measured fibre orientations reads as follows: