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Free-form optimization of sandwich structures for controlling thermal displacement

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ABSTRACT

Considering the different thermal expansion coefficients of skins and cores, sandwich structures whose design optimization is performed have the potential for application in thermal actuators. In the present work, we propose a free-form optimization method for the design optimization of sandwich structures composed of two thermoelastic materials (*A* and *B*) for controlling their thermal displacements. We use the square displacements error norm between the thermal displacement and target displacement at the designated boundary as the objective function, and minimize it by optimizing the interface and the surface shapes of the sandwich structures under thermal load. We derive the shape gradient function by using the material derivative method and apply it to the free-form optimization method for determining the optimal shape. Two volume constraints, the total volume and volume of material *A*, are considered in the problem formulation. The numerical results show that the thermal deformation of the optimal sandwich structure can be well identified; this is helpful for applying sandwich structures in thermal actuators.

1. Introduction

Sandwich structures, which are a special class of composite structures, are fabricated by attaching a pair of skins and a core for realizing high mechanical behaviors with low overall density. They have been widely used in fields, such as aerospace, automotive, energy, and civil engineering. Karlsson and Åström [1] summarized most of the techniques employed to manufacture sandwich structures and the applications of structural sandwich components, and discussed the recent developments and future trends at that time. The properties of such sandwich structures depend largely on the outer skins and the interface between the skins and the core. The materials used as cores in sandwich structures include honeycomb (e.g., nomex and polypropylene), aluminum, balsa wood, and cellular foams, for weight reduction, impact resistance, thermal insulation and so on [2–4]. Recently, Grünewald et al. [5] reviewed the manufacturing of thermoplastic sandwich structures by using skin-core joining methods, and pointed out that the joining of skins and core is a critical step in the manufacturing of sandwich structures.

The mechanical behaviors of sandwich structures (e.g., bending, vibration, and buckling properties) have been investigated experimentally and theoretically [6–9]. Thomsen [6] presented a method to estimate local bending effects in a circular sandwich plate subjected to a central point load by combining theoretical analysis and experiments; the local bending phenomenon could be described well by this method. By applying von Karman-type geometric nonlinearity and the first-order shear deformation plate theory, Yang et al. [7] investigated the nonlinear local bending behavior of a rectangular sandwich plate with a functionally graded (FG) core. A comparison of its performance with that of a sandwich plate with a homogeneous core showed that the FG core effectively reduced both the local deformation and interfacial shear stresses. Lou et al. [8] studied the effects of local damage on the vibration behavior of sandwich structures with pyramidal truss cores by applying the finite element method (FEM) for an experimentally validated model. All of these works showed that sandwich structures have important advantages in terms of high strength and stiffness, good vibration damping, and so on.

Sandwich structures also have an advantage in terms of their thermoelastic behavior. For the thermal stress analysis of sandwich beams, Kapuria et al. [10] proposed a new higher order zigzag theory, which is generally more accurate and shows a better performance than the previous zigzag theory. Zenkour and Alghamdi [11,12] carried out thermoelastic bending analyses of FG sandwich plates and concluded that the inclusion of transverse normal strain in the analysis can increase the shear stress and

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decrease the deflection and axial stress. In addition, the thermoelastic analysis of sandwich structures with FG [13], truss [14], honeycomb [15], foams [16,17], and lattice [18] cores were conducted based on theoretical and experimental approaches.

Bimaterial thermal actuators were designed for based on the differential thermal expansion of distinct materials [19–21]. Owning to the thermoelastic characteristic of sandwich structures that skins and cores have different thermal expansion coefficients, sandwich structures also have potential for application in thermal actuators if the shapes of skins and cores are appropriately designed. Therefore, the design optimization of sandwich structures should be carried out for controlling their thermal displacements. In the past decade, design optimization of such structures has progressed in a variety of research directions. Denli and Sun [22,23] studied the structural–acoustic optimization of sandwich structures for realizing minimum sound radiation. Awad et al. [24] reviewed the design optimization techniques of the fiber composite structures used in civil engineering at that time. Lee et al. [25] optimized sandwich cylinders under external hydrostatic pressure for underwater vehicle applications. Based on the Direct MultiSearch method, Araújo and Madeira et al. [26,27] carried out design optimization of sandwich plates for active damping and vibration reduction. Furthermore, the design optimization of various types of sandwich structures, such as foam-reinforced corrugated sandwich structures [28] and sandwich structures with honeycomb [29] and porous cellular [30] cores, were also studied recently.

Despite the effort expended on design optimization, an efficient and effective method for controlling the thermal displacements of sandwich structures is yet to be developed. This study focuses on the design optimization of sandwich structures for controlling their thermal displacements. In our previous work [31], we proposed a gradient optimization method for designing the interface shape of sandwich structures to minimize their compliance. In the present work, we aim to further develop this optimization method for the shape identification design problem of sandwich structures under thermal load. The outline of this study is as follows. Sections 2 and 3 explain the domain variation and formulation for controlling the thermal deformation of sandwich structures, respectively. The derivation of the shape gradient density functions based on the material derivative method is described in Section 3. In Section 4, we introduce the free-form optimization system based on the gradient method in the H^1 function space. Section 5 provides three numerical examples for verifying the validity of the proposed optimization method. Finally, the conclusions are summarized in Section 6.

2. Domain variation of sandwich structures

We first introduce the domain variation of sandwich structures. A sandwich structure composed of materials A and B with an initial global domain $\Omega \subset \mathbb{R}^3$ and boundary $\Gamma \equiv \partial\Omega$ is shown in Fig. 1. Here $\mathbb{R}$ indicates a set of positive real numbers. The sub-domains of materials A and B are Ω_A and Ω_B, respectively, and their boundaries are Γ_A and Γ_B, respectively. The interface between Ω_A and Ω_B is $\Gamma_{AB} \equiv \Gamma_A \cap \Gamma_B$. Further, Γ_D is the designated boundary of thermal deformation. In a variation design velocity field $\boldsymbol{V}$, the domain and boundary of the sandwich structure become Ω_s and Γ_s, respectively. Hence, the domain variation can be expressed by a one-to-one mapping $T_s(\boldsymbol{X}) : \boldsymbol{X} \in \Omega \mapsto \boldsymbol{x} \in \Omega_s,\ 0 \leqslant s < \varepsilon$, where s and ε indicate the iteration history (or the design time) of domain variation and a small positive number, respectively. We assume a constraint acting on the domain $\Theta \subset \Omega$. The infinitesimal domain variation of the sandwich structure can be given as follows:

$$T_{s+\Delta s}(\boldsymbol{X}) = T_s(\boldsymbol{X}) + \Delta s \boldsymbol{V} \tag{1}$$

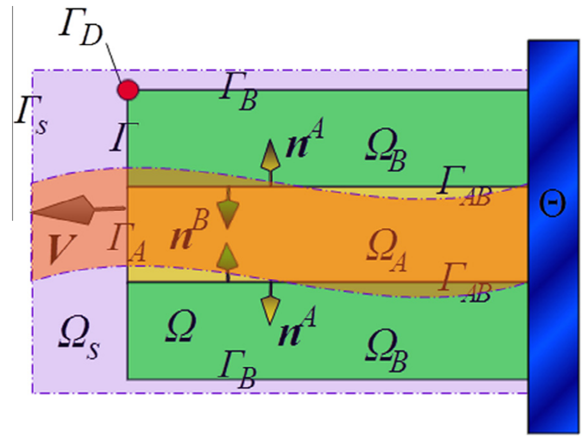

Fig. 1. Domain variation of a sandwich structure.

where Δs is the amount of domain variation in s as determined from a trial implementation. The design velocity field $\boldsymbol{V}$ is a piecewise continuous function, which contains a derivative of $T_s(\boldsymbol{X})$ in terms of s.

$$\begin{aligned}\boldsymbol{V}(\boldsymbol{x}) &= \frac{\partial T_s}{\partial s}(T_s^{-1}(\boldsymbol{x})), \quad \boldsymbol{x} \in \Omega_s, \quad \boldsymbol{V} \in C_\Theta \\ &= \left\{\boldsymbol{V} \in C^1(\Omega; \mathbb{R}^3) | \boldsymbol{V} = \boldsymbol{0} \text{ in} \Theta\right\}\end{aligned} \tag{2}$$

where C_Θ is the suitably smooth function space that satisfies the constraints of domain variation. The optimal design velocity field $\boldsymbol{V}$ in Eq. (2) can be determined by the free-form optimization method, which will be introduced in Section 4.

3. Formulation of controlling thermal deformation of sandwich structures

In the design optimization for controlling the thermal deformation of sandwich structures, we consider a shape identification design problem by using the squared error norm between thermal displacement $\boldsymbol{v}$ and the target displacement $\hat{\boldsymbol{v}}$ at Γ_D as the objective function (Eq. (5)). We minimize this function under two volume constraints (Eq. (6)) and by using the governing equation (Eq. (7)). The design optimization problem is formulated as follows:

$$\text{Given} \quad \Omega \tag{3}$$

$$\text{find} \quad \boldsymbol{V} \tag{4}$$

$$\text{that minimizes} \quad d(\boldsymbol{v} - \hat{\boldsymbol{v}}, \boldsymbol{v} - \hat{\boldsymbol{v}}) \tag{5}$$

$$\text{subject to} \quad M\left(= \int_\Omega d\Omega\right) = \hat{M}, \quad M_A\left(= \int_{\Omega_A} d\Omega\right) = \hat{M}_A \tag{6}$$

$$\begin{aligned}\text{and} \quad & a_A(\boldsymbol{v}, \boldsymbol{w}) - h_A(\boldsymbol{v}, \boldsymbol{w}) + a_B(\boldsymbol{v}, \boldsymbol{w}) - h_B(\boldsymbol{v}, \boldsymbol{w}) \\ &= l_A^\theta(\boldsymbol{w}) + l_B^\theta(\boldsymbol{w}), \quad \forall \boldsymbol{w} \in U, \quad \boldsymbol{v} \in U\end{aligned} \tag{7}$$

where the squared error norm of thermal displacements $d(\boldsymbol{u}, \boldsymbol{v})$ is defined as follows:

$$d(\boldsymbol{u}, \boldsymbol{v}) = \int_{\Gamma_D} u_i v_i d\Gamma \tag{8}$$

M and M_A given in Eq. (6) are the total volume and the volume of material A, respectively. The notation $(\hat{\ })$ denotes the constraint values. Eq. (7) is the weak-form governing equation of the thermal deformation of sandwich structures, where $\boldsymbol{v}$ and $\boldsymbol{w}$ are the displacement vector and the variational displacement vector,