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Two-dimensional modal curvature estimation via Fourier spectral method for damage detection

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ABSTRACT

Modal curvature is one of the most important damage indices utilized in the damage identification for composite structures. However, few kinds of sensors can provide the measure for modal curvature directly. The lack of direct measurement method for modal curvature necessitates the use of the central difference estimation, which reduces the stability of the algorithm. Noise is severely amplified by the central difference, and hence limits the damage identification through the use of modal curvature estimation. Instead of numerical differentiation, the two-dimensional Fourier spectral method is employed to conduct the two-dimensional modal curvature estimation in this paper. The use of Fourier spectral method over the conventional central difference operator gives the proposed methodology the following advantages: (1) Spectral calculations for spatial derivatives are implemented in global space, thus noise can be suppressed. The *k*-domain generated in algorithm provides the space for spatial filtering. (2) The precise estimation for modal curvatures can be obtained by aid of the trigonometric interpolation in spectral method. (3) The proposed method calculates the modal curvatures based on fast Fourier transform, thus the efficiency is ensured. By the numerical and experimental comparisons with the classical central difference method, the suitability of the present method is verified for composite structures.

1. Introduction

The investigations of the dynamics features for two-dimensional structures have drawn more and more attentions [1–4]. In the last few decades, the vibration-based structural damage detection has been an area of active research in various fields for its on-line characteristic. The variations of natural frequencies are employed as the damage index in majority of approaches as: frequencies are easily measured with high accuracy. However, the lumped quantities of frequencies are less sensitive to damage location [5–9]. In comparison, mode shape (containing its derivatives) provides abundant damage localization information [10–19]. It points out that Xiang et al. [19] proposed a two-step method combining curvature mode shape and wavelet finite element model-based method to determine damage locations and severities.

The changes in modal displacement between the intact and damaged structures have been exploited in damage detection over the past years. The major limitation is that, even if the damage is localized, damage effect on modal displacement is spread all over the structure [10]. As a consequence, identification may be cumbersome when several damages occur simultaneously. On the contrary, the use of modal curvatures (MC) generates a superior indicator for damage localization. The comparative study given by Fan and Qiao [20] shows that MC methods are more robust and promising compared with the modal displacement method when MC is directly measured from the strain mode shape. Given that the displace ment/velocity/acceleration data are easily measured and widely used in application, the numerical estimation for MC is still the mainstream now. The well-known central difference is one of the most popular approaches in the numerical estimation of MC. In the one-dimensional case, such as rod, beam and shaft, MC can be written as:

$$w''(x) = \frac{d^2w(x)}{dx^2} = -\frac{M}{EI(x)} \quad (1)$$

where $w(x)$ is the modal displacement, $w''(x)$ is the one-dimensional MC, $W(x)$ is the modal displacement, M is the bending moment, and $EI(x)$ is the bending stiffness. According to central difference method, Eq. (1) is rewritten as:

$$w''(x) = \frac{w(x-\Delta) - 2w(x) + w(x+\Delta)}{\Delta^2} \quad (2)$$

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where Δ is the distance between two adjacent points used in estimation.

Eq. (1) can be extended to plates and shells. The two-dimensional MC, denoted as MC^{2D}, can be represented as [21]:

$$w''(x,y) = w''_x + w''_y = \frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2} \tag{3}$$

where

$$\frac{\partial^2 w(x,y)}{\partial x^2} = -\frac{M_x(x,y)}{D_0}, \quad \frac{\partial^2 w(x,y)}{\partial y^2} = -\frac{M_y(x,y)}{D_0} \tag{4}$$

$$D_0 = \frac{Eh^3}{12(1-\mu^2)} \tag{5}$$

$M_x(x,y)$ and $M_y(x,y)$ are the moments, and μ is the Poisson's ratio. The reduction of D_0 is induced by the presence of damage in plates. By aid of central difference, MC^{2D} is approximated as:

$$w''_{x,y} = \frac{w(x-\Delta_x) - 2w(x) + 2w(x+\Delta_x)}{\Delta_x^2} + \frac{w(y-\Delta_y) - 2w(y) + 2w(y+\Delta_y)}{\Delta_y^2} \tag{6}$$

In the representation $w''_{x,y}$, subscript is utilized to emphasize that the left side of Eq. (6) is the estimation of $w''(x,y)$. In the sensitivity investigation about the static and dynamic features of damaged plates, Yam et al. [22] proposed to use the difference between the intact MC^{2D} and the damaged MC^{2D} as damage index. Qiao et al. [20,23] investigated the application of MC^{2D} in the delamination detection for composite laminated plates. Wu and Law [24,25] developed a method to use the curvature of the uniform load surface resembling the MC^{2D} for damage detection in plates. An ameliorated MC^{2D}, gapped smoothing method, was proposed by Yoon et al. [26,27] for damage detection in plates. These methods are capable of detecting damage in two-dimensional structures.

Despite the MC^{2D} estimated by Eq. (6) has been proven to be a good damage index, it still has the notable deficiency, the susceptibility to noise [16,28–31]. To overcome this problem, many excellent investigations have been conducted by aid of Teager energy operator, wavelet and hybrid methods [15,21,32–34]. Motivated by these investigations, the alternative and simple modal curvature estimation via Fourier spectral method is proposed in this paper with the following superiorities:

(1) The differentiation of functions can be easily conducted via multiplication in the k-domain (wavenumber domain) of the Fourier transform (FT), and reconstruction via the inverse Fourier transform (IFT).
(2) The approximation function in Fourier spectral is trigonometric function. Thus, the approximating accuracy is relatively high.
(3) Noise can be suppressed via averaging in the k-domain (k-domain). Fourier spectral-based methods are global, thus the parts of the noise can be suppressed in k-domain via spatial filters.

This paper is organized as follows: Section 2 presents the formula and implementation of the Fourier spectral method for MC^{2D} estimation. Section 3 presents the comparative and numerical studies for the proposed method. Section 4 validates the proposed method further via benchmarks for composite structures. In the end, some discussions and conclusions are presented in Section 5.

2. Fourier spectral method for MC estimation

2.1. Theoretical basis

The application of Fourier spectral methods to solve partial differential equations can be traced back to Lanczos in 1938 [35]. Until 1970s, a transform of field was initiated by Orszag [36], this method became famous. Now, it has been a popular numerical method in various fields with its wavelet and pseudo spectrum extensions. In this paper, we apply the two-dimensional Fourier spectral methods to estimate the MC^{2D}. For the two-dimensional mode shape $w(x,y)$, the FT and the corresponding inverse FT (IFT) are written as:

$$w(x,y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ik_x x}\hat{w}(k_x,y)dk_x, \quad w(x,y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ik_y y}\hat{w}(x,k_y)dk_y \tag{7}$$

$$\hat{w}(k_x,y) = \int_{-\infty}^{+\infty} e^{-ik_x x}w(x,y)dx, \quad \hat{w}(x,k_y) = \int_{-\infty}^{+\infty} e^{-ik_y y}w(x,y)dy \tag{8}$$

where $x,y \in \mathbb{R}$ and $k_x,k_y \in \mathbb{R}$. k is called wavenumber with the specific physical meaning. In FT, derivation can be expressed as the combination of multiplication and IFT as:

$$\frac{\partial w(x,y)}{\partial x} = \frac{1}{2\pi} i \int_{-\infty}^{+\infty} k_x e^{ik_x x}\hat{w}(k_x,y)dk_x \tag{9}$$

$$\frac{\partial w(x,y)}{\partial y} = \frac{1}{2\pi} i \int_{-\infty}^{+\infty} k_y e^{ik_y y}\hat{w}(x,k_y)dk_y \tag{10}$$

According to Eqs. (9) and (10), one can get:

$$\frac{\partial^2 w(x,y)}{\partial x^2} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} k_x^2 e^{ik_x x}\hat{w}(k_x,y)dk_x \tag{11}$$

$$\frac{\partial^2 w(x,y)}{\partial y^2} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} k_y^2 e^{ik_y y}\hat{w}(x,k_y)dk_y \tag{12}$$

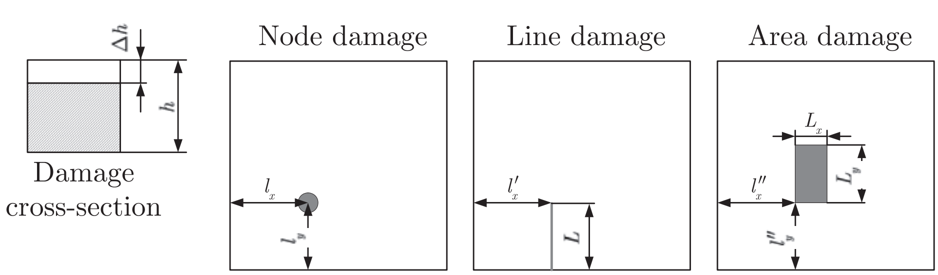

Fig. 1. The illustrations of damages.